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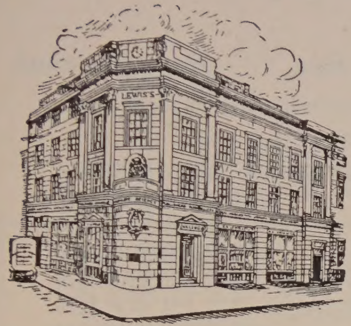

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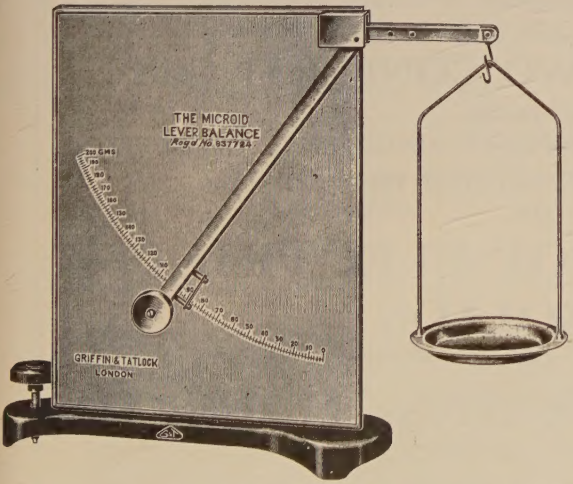

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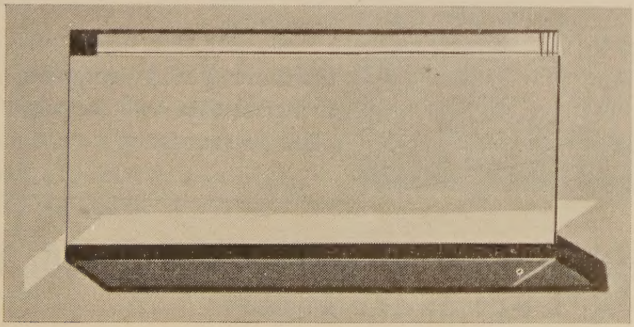

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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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A NEW TREATMENT OF ELECTRIC AND MAGNETIC INDUCTION

By G. BURNISTON BROWN, M.Sc., Ph.D.,
University of London, University College

Received 4 *January* 1940. *Read* 3 *May* 1940

ABSTRACT. A new treatment of induction is suggested which, by considering "theoretical-experiments" and the meaning of the terms "force on a pole" and "force on a current element", arrives at the definition of the internal field and the induction without involving an arbitrary selection of certain cavities.

It is shown that the dielectric constant and the permeability are strictly analogous measures of the effects of polarization, and a definition of the permeability is given which resembles the definition of the dielectric constant.

A very simple derivation of the formula known as the Clausius-Mosotti formula is given, and what would appear to be errors in the Mosotti-Jeans and Clausius-Lorentz treatments are pointed out. The use of the latter formula, it is suggested, may lead to errors in the calculated dipole moment.

A proof is then given that the induced electromotive force depends on the change of induction and not on the change of force: this is rarely, if ever, proved satisfactorily in the text-books. A method of obtaining Maxwell's equations without invoking displacement currents is indicated.

The effect of a polarizable medium on the forces between point charges, point poles, point poles and current elements, current circuits and magnetic shells, and current circuits and actual magnets, is examined in detail. The five electromagnetic relations are re-stated with the effect of the medium explicitly mentioned, and the dimensionless nature of the dielectric constant and the permeability is stressed.

---

§ 1. INTRODUCTION

It is well known that in classical electromagnetic theory the treatment of phenomena occurring, or supposed to occur, in magnetizable media is clouded with considerable obscurity, amounting in some cases to actual contradiction. And it cannot be said that the decisions of the Committee of the International Electrical Congress (arrived at by a majority vote of 9 to 3), which resulted in the introduction of the Oersted, have done more than add to the confusion. Indeed the secretary of the committee, Prof. Henri Abraham, maintains that there are at least half a dozen definitions of the permeability μ, and he concludes that their equivalence is not a matter to be solved by experiment, but is " une question d'opinion métaphysique " (1).

Every student of this subject has painful memories of certain cavities which are introduced arbitrarily, one of which is used to define the magnetic force and the other the magnetic induction. For some reason, never clearly explained, a current circuit is affected by the induction and not by the magnetic force. Further, in the anchor-ring experiment the student is unable to discern why the throw of his ballistic galvanometer should depend on the force in a cavity of one shape rather than another, especially in view of the fact that the anchor-ring *contains no cavities at all.*

Again, a serious contradiction appears to arise when the physical equivalence of magnetic shells and current circuits embedded in a magnetizable medium is considered. According to several well-known text-books the effect of immersion in a medium of permeability μ is

(1) to *decrease* the force between two magnetic poles in the ratio $\mu : 1$,
(2) to leave the force between currents and magnetic poles unaffected,
(3) to *increase* the force between two currents in the ratio $\mu : 1$.

The last effect is not usually stated explicitly but follows logically from (1) and (2). It is given in the earlier editions of Starling's well-known *Electricity and Magnetism* (Longmans) but it has disappeared from the latest edition. Now it is clear that if we believe, as we have done from the time of Ampère himself, that all magnetism is electromagnetism, that is to say, that all magnetic phenomena are the effects of motion of electric charge, then a contradiction arises if we assert that the effect of a magnetizable medium on the field due to a magnet is different from its effect on that due to an electric current.

§2. ON THE USE OF THEORETICAL-EXPERIMENTS

In attempting to resolve confusion in the logical development of a physical theory we must examine the definitions of the concepts involved and the experiments or "theoretical-experiments" through which they receive their precise meaning. And we must take care that even in a theoretical-experiment we do not assume a process that is contrary to natural laws. Absolute time, absolute space, and absolute simultaneity have disappeared from physical theory because they could not be given a precise meaning in an experiment or a theoretical-experiment. The possibility of knowledge of the position and velocity of an atomic particle at one and the same instant has disappeared from physical theory because in the theoretical-experiment it was assumed that the disturbance of the particle caused by the act of observation could be diminished indefinitely. This has turned out to be contrary to natural law. What has been said above is now accepted by physicists and no further justification is required here. We may therefore proceed to enquire what is the meaning of the words " force in a polarizable medium ".

For "polarizable" we can use Maxwell's very general definition, viz. "an elementary portion of a body may be said to be polarized when it acquires equal and opposite properties on two opposite sides". The treatment will then apply, if we restrict ourselves to *induced* magnetism, to dielectric as well as magnetic media, which is essential in any attempt to show that the dielectric constant K and the permeability μ, which occupy analogous positions in Maxwell's electromagnetic equations, are actually analogous in derivation.

§3. RESTRICTION TO INDUCED MAGNETISM IN ISOTROPIC MEDIA

The restriction introduced above, viz. to the phenomena of induced magnetism in isotropic media, amounts to saying that we shall deal only with cases which are of fundamental importance in electromagnetic theory. The fields inside permanent magnets in which the magnetization is independent of the given magnetizing field, and media which are not isotropic, are of little importance and can be treated as special cases. The definitions of K and μ are such that they only apply to phenomena in which the polarization is the result of the polarizing field.

§4. DEFINITION OF THE INTERNAL MAGNETIC FORCE (H) AND THE INDUCTION (B)

Force is the hypothetical cause of change, and in classical physics it is made precise and measurable by the convention expressed in Newton's second law, that

$$\text{force} = \text{mass} \times \text{acceleration}.$$

We must therefore measure the acceleration, and in a medium this will require a cavity with finite extension in the direction of the force which acts on the mass in question. It should be noted that even if a null method is adopted, involving some kind of balance, it is still necessary to have some free space in the direction of the force, for the balance to take up the position of equilibrium. No inference can be made from a null method unless it is shown that a change in the quantities concerned prevents the null reading. This was pointed out long ago by Poynting in connexion with the inference, often made, that because there is no force inside a hollow conductor, therefore the law of force is that of the inverse square(2).

Now when we are dealing with an electric charge or magnetic pole in a theoretical-experiment we may suppose the charge or pole as small as we please, but we must have finite extension in the direction of the force on the charge or pole. To measure this force we must suppose the pole* to be placed inside the medium, where it will occupy a very small spherical volume which we must suppose to be evacuated. If equilibrium does not exist, the pole will tend to move in a certain direction, and in order to measure the acceleration free from any resistance of the medium we imagine the medium removed in this direction so that we have a cylindrical cavity with its axis parallel to the force. The diameter of the cylindrical cavity may be as small as we please, since we are dealing with imaginary point poles, but the length must be finite. Unless the force is in the direction of magnetization, however, the cutting of the cavity will cause a surface distribution of pole to appear on the walls as well as on its ends, and thus the cavity necessary for the act of measurement will alter the force to be measured. The force, therefore, is not in general determinate. This difficulty can be overcome

* The argument is restricted to poles for convenience.

in two ways: one way is that adopted by Maxwell, viz. to define the shape of the cavity arbitrarily and to make the direction of its axis that of the magnetization; the force measured inside may then have *any* direction. This results in an unambiguous definition of the force, but leads to obscurity as regards its physical significance. The second way, which will be adopted here, is to restrict our consideration to cases in which the pole tends to move in the direction of the magnetization. This will be the case inside an ellipsoid of isotropic material placed in a uniform external magnetizing field, as was shown by Kelvin⁽³⁾. Since an ellipsoid can be made to approach both the shape of a thin rod and a flat disc, it is sufficient for our purpose, viz. the consideration of forces within long thin rods in solenoids and anchor-rings, and inside slabs of material between parallel-plate condensers or the poles of electromagnets.

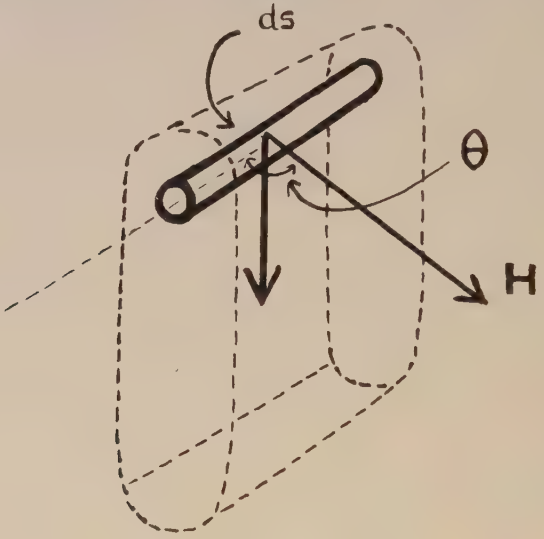

Figure 1.

We can therefore define the force along the axis in such a cavity as we have described as the *Internal Magnetic Force* (*H*), or in the electrical case, the *Internal Electric Force* (*E*). This cavity will be referred to as the *rod-shaped cavity*, and it should be noted that the strength of the charge or pole used for measuring the force within it is assumed so small that it does not itself polarize the medium.

Leaving electrostatics and magnetostatics and concentrating on electromagnetism, let us consider what meaning can be attached to the concept of "force on an element of conductor carrying a current". We notice at once that experiment shows two important differences from the case of a magnetic point pole. Firstly, the force on the element of conductor is at right angles to the direction of the magnetic field instead of being along it, and secondly, the force depends on the direction of the current and consequently the element of conductor cannot be treated as a point but must have finite extension in the direction of the current.

To measure this force we suppose the element ds of conductor carrying a current i to be placed in the medium where it will occupy a small evacuated cylindrical volume making an angle θ, say, with the direction of the magnetization (figure 1). Any force on it will cause it to move in a direction which will be perpendicular both to the element itself and to the magnetic field, and in order to measure it we imagine the medium removed in the direction of the force so that we have a thin rectangular-shaped cavity, one side being finite and parallel to the element of conductor, another finite and parallel to the direction of the force, and the third, perpendicular to the other two, can be made as small as we please.

We could call this force per unit current per unit length of conductor the *Internal Electromagnetic Force*. It would be zero when the element of conductor was parallel to the Internal Magnetic Force and a maximum when perpendicular to it.

In order, however, to conform with the classical treatment we proceed as follows:

The force on the conducting element in air (more strictly in a vacuum) we know from Ampère's experiments and Laplace's theory can be written

$$\text{Force on conductor} = i\, ds H_A \sin\theta,$$

where $H_A \sin\theta$ is the component normal to ds of the magnetic force H_A in air, and the direction of the force on the conductor is perpendicular to the direction of H_A.

In the cavity let us now write in a similar manner

$$\text{Force on conductor} = i\, ds B \sin\theta.$$

We can then define $B \sin\theta$ as the component normal to ds of the *Induction* (B).

We are therefore led by the examination of the meaning to be attached to the expression " force in a medium " to the consideration of the force in certain cavities. In order to make calculation easier and to avoid " edge-effects " we now imagine the cavities enlarged (while still remaining indefinitely thin) so that the force measurement takes place at their centres only. It is then clear that we could substitute a circular disc-shaped cavity for our rectangular slot without affecting the force at the centre. We then recognize the characteristic and arbitrarily introduced cavities of the Kelvin-Maxwell treatment.

In order to conform with present universally-adopted treatment we now make the following definitions:

The Internal Magnetic Force (H) is the force per unit pole on a magnetic north pole placed at the centre of an evacuated rod-shaped cavity whose direction is that of the magnetization.

The Induction (B) is the force per unit pole on a magnetic north pole placed at the centre of an evacuated disc-shaped cavity whose axis is in the direction of magnetization. This force is equal in magnitude but perpendicular in direction to the maximum Internal Electromagnetic Force.*

---

* Kelvin proposed to call this " the electromagnetic definition of magnetic force ".

§ 5. DIFFICULTIES INHERENT IN THE NOTION OF A FIELD OF FORCE

It is perhaps useful at this point to consider a difficulty which is inherent in Faraday's view that the seat of electromagnetic action lies in the space surrounding bodies—the "field of force". This difficulty arises because we have to suppose a magnetic field to act with one force and one direction on a pole and another force and another direction on a current element (the magnitude of the force is made the same in air by arbitrary definition of unit current) which amounts to having two different fields at one point. We are not content to treat the problem as one of interaction between different objects, which is what we observe, but prefer to consider it as interaction between different objects and space. This leads to the question whether the two fields in space are really different, and whether one should be measured in gauss and the other in oersteds.

This failure of the field-of-force idea has been hidden, in the past, by the arbitrary cavity treatment of Kelvin and Maxwell and their emphasis on a mathematical aspect, viz. that the induction is a solenoidal vector. The present treatment attempts to show how the Kelvin-Maxwell cavities arise naturally instead of arbitrarily if the measuring operations by which we attach numbers to the Newtonian force-concept are carefully kept in mind. The definition of the induction in terms of the force on a magnetic pole rather than on a current element has been made to conform with classical presentation.

§ 6. THE DETERMINATION OF THE RELATION BETWEEN THE VALUES OF THE INTERNAL MAGNETIC FORCE AND THE INDUCTION AND THE DEFINITION OF THE SUSCEPTIBILITY κ AND THE PERMEABILITY μ

We must now proceed to derive expressions for the force in the cavities we have chosen. For this purpose we make use of a theorem due to Poisson which is based solely on the inverse-square law of force, the fact that when a magnetized substance is cut, new magnetic poles appear on the surfaces created by the cutting, and the assumption that the field of a magnetized body can be treated as consisting of the fields of a large number of doublets. Our argument will then be a macroscopic one and independent of any theories as to the relation between molecular structure and magnetic properties. This is a great advantage, and further, means that the treatment is equally applicable to dielectrics. In this connexion, however, we must ensure that the cavities are so small that the intensity of magnetization (I) does not vary appreciably in their neighbourhood, and yet at the same time, they are large compared with intermolecular distances. That this is the case in practice is shown by Jeans's argument:

"In steel or iron there are roughly 10^{23} molecules to the cubic centimetre, so that a length of 1 millimetre may be regarded as large when measured by the molecular scale, although in most magnets the magnetization may be treated as constant within a length of a millimetre."(4)

Now following Poisson it can be shown that in a cavity inside a magnetized body the force can be considered as the resultant of four forces:

(1) The force due to magnets or currents external to the body concerned. This force might be modified by the resulting magnetization produced, but in practice we deal with the fields of permanent magnets, electromagnets and currents, and these are not affected by the magnetization they produce. If we confine ourselves to such cases, we can call this force H_A, and it is measured as the force per unit north pole in air (strictly, *in vacuo*) at the point considered, the body being removed.

This is the *Magnetizing Force* (H_A) and the following three forces are the result of its polarizing action on the particles of the body;

(2) A force F_1 due to a distribution of surface density $(lA+mB+nC)$ spread over the external surface of the body, where A, B and C are the components of the Magnetization (I) parallel to the axes of x, y and z and l, m, n are the direction cosines of the outward-drawn normal to the element of surface. I is, of course, the magnetic moment per unit volume ;

(3) A force F_2 due to a distribution of surface density $(lA+mB+nC)$ spread over the walls of the cavity ;

(4) A force F_3 due to a distribution of volume density

$$-(\partial A/\partial x+\partial B/\partial y+\partial C/\partial z)$$

spread throughout the whole space which is occupied by the body after the cavity has been made.

As regards the last force F_3, if we restrict ourselves, as we have done, to isotropic media and to cases in which the magnetization is produced solely by the given magnetic field (just as the polarization of a dielectric is produced by the electric field), then the magnetization of any small volume of the medium is always in the direction of the force existing in that small volume and is proportional to it, so that we have

$$I=\kappa H, \text{ where } \kappa \text{ is a constant,}$$

and $$A=-\kappa\partial\Omega/\partial x, \quad B=-\kappa\partial\Omega/\partial y, \quad C=-\kappa\partial\Omega/\partial z,$$

where Ω is the magnetic potential in the small volume, so that Poisson's volume distribution becomes

$$\rho=-(\partial A/\partial x+\partial B/\partial y+\partial C/\partial z)=\kappa(\partial^2\Omega/\partial x^2+\partial^2\Omega/\partial y^2+\partial^2\Omega/\partial z^2).$$

But the potential in the small volume is the superposition of that due to the external field and that of a large number of doublets. For each of these $\nabla^2\Omega=0$, so that

$$\rho=0$$

and the force that we have called F_3 is zero.

The force F_1 acts so as to diminish H_A, the Magnetizing Force, and is called the *Demagnetizing Force*. If we take H_A positive, then F_1 is negative and F_2 positive. The force in any cavity is therefore the resultant of

$$H_A-F_1+F_2 \qquad \ldots\ldots(1).$$

Now in a rod-like cavity with its axis parallel to the direction of magnetization, the distribution $(lA+mB+nC)$ vanishes over the sides, and is equal to $\pm I$ over the two ends. The effect of this distribution on the end faces becomes negligible as the ratio of the diameter to the length of the cavity is decreased. This ratio can be made as small as we please for a point pole, and so the force F_2 is zero in a long narrow rod-like cavity and we are left with

$$H_A - F_1,$$

and this we have called the Internal Magnetic Force H, so that

$$H = H_A - F_1 \qquad \ldots\ldots(2).$$

The ratio of the Intensity of Magnetization to the Internal Magnetic Force is defined as the *Susceptibility* κ,

$$I/H = \kappa.$$

In several ways it might appear better to define the susceptibility as the ratio of the Intensity of Magnetization to the Magnetizing Field I/H_A (indeed most text-books do so define it, but they write I/H). H_A is a measure of the cause and I a measure of the effect, whereas $H = H_A - F_1$ is partly cause (H_A) and partly effect ($-F_1$). Further, H_A is directly measurable whereas H is not. However, it will be shown later that if we wish to have the permeability μ strictly analogous to the dielectric constant K and also the relation $\mu = 1 + 4\pi\kappa$ we are forced to choose I/H as the definition of susceptibility. Nevertheless confusion is sometimes caused: Pidduck, for instance, describes an experimental method for determining the susceptibility of an iron wire which consists in finding the ratio I/H_A(5).

Turning now to the disc-shaped cavity, we find in the usual manner that the Induction is

$$H_A - F_1 + 4\pi I$$

or $$B = H + 4\pi I \quad \text{from (2)}.$$

Let us now define the permeability as the ratio B/H, or, more interestingly, as the ratio of the maximum Electromagnetic Force to the Magnetic Force in the medium. Then we have

$$B = \mu H = H + 4\pi I$$

or $$\mu = 1 + 4\pi\kappa \qquad \ldots\ldots(3)$$

in the usual manner.

We have now to show that μ defined in this way is the analogue of K defined as the ratio of the capacity of a condenser containing the medium as dielectric to the capacity of the same condenser with air (strictly a vacuum) as dielectric. Consider figures 2 (*a*) and 2 (*b*): in figure 2 (*a*) we have an insulated and charged parallel-plate condenser with a slab of dielectric constant K between the plates, and in figure 2 (*b*) we have a gap between the poles of a magnet which is occupied by a slab of permeability μ. The internal force will be in each case, as we have seen, the force at the centre of a rod-like cavity with its axis parallel to the field, which is uniform if the cavity is far from the edges. The intensity of polarization will be uniform also, and so we can, if we like, imagine the cavity extended right across the medium so that the charge or pole can be drawn through from one side to the other.

F_2 is zero in this shape of cavity, and so we have for the internal forces

$$E_A - 4\pi\sigma \quad \text{and} \quad H_A - 4\pi I,$$

where E_A corresponds to H_A and is the external electric field supposed unaltered by the resulting polarization and σ corresponds to I and is the electric moment per unit volume (or the surface density).

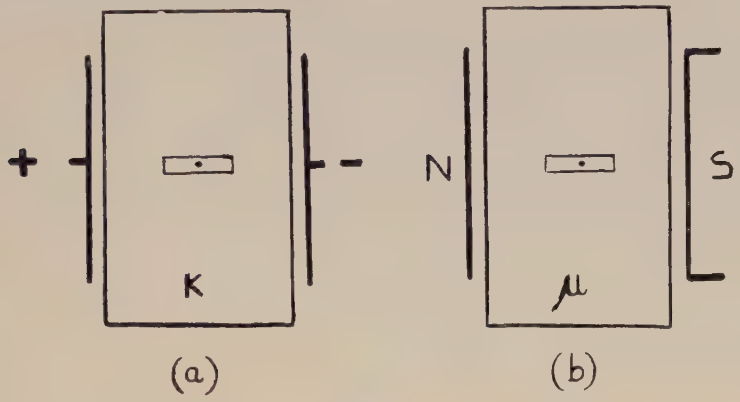

Figure 2.

Now in electrostatics we infer from experiment that the potential of the condenser drops to $1/K$ of its value in air on substituting the medium of dielectric constant K. Consequently we must suppose the force to be diminished also in this ratio, and so we write

$$E_A - 4\pi\sigma = E_A/K \qquad \text{......(4),}$$

and if we do the same in the magnetic case we get

$$H_A - 4\pi I = H_A/\mu \qquad \text{......(5).}$$

Now, $H = H_A - F_1 + F_2$ from (1) and $F_2 = 0$, $F_1 = 4\pi I$, so that

$$H_A = H + 4\pi I$$

and (5) becomes

$$\mu H = H + 4\pi I \qquad \text{......(6)}$$

or

$$\mu = 1 + 4\pi\kappa,$$

which agrees with (3).

We have, therefore, shown that μ defined as equal to B/H is the analogue of K in electrostatics. Various formulae derived in electrostatics can therefore be transferred to magnetism, e.g. the energy per unit volume will be $\mu H^2/8\pi$ and Gauss's theorem becomes

$$\iint \mu H \cos\theta \, ds = 0,$$

since Σm, the sum of the pole strengths, is always zero. Thus B is a solenoidal vector.

The error in taking $\kappa = I/H_A$ instead of $\kappa = I/H$ can easily be shown to be of the order of 1 per cent if κ is about 1×10^{-3}. Curie's well-known method of determining κ involving a non-uniform field measures I/H_A and is therefore only suitable for paramagnetic and diamagnetic bodies whose susceptibility does not exceed the value given.

§7. THE MOSOTTI-JEANS AND CLAUSIUS-DEBYE FORMULAE

Let us re-write (4) so that it resembles the magnetic formula (6):

$$KE = E + 4\pi\sigma$$

or

$$K = 1 + \frac{4\pi\sigma}{E}.$$

Now let us amplify our macroscopic treatment, remembering that it is based on the superposition of the fields due to doublets. Suppose there are n doublets per cm^3 and that each is capable of being polarized so that the electric moment ϵ is proportional to the electric field in which it lies. Then for ellipsoidal isotropic bodies, in a uniform field,

$$\sigma = n\epsilon = n\alpha E,$$

where α is the *Polarizability* of the particle ($n\alpha$ corresponds to κ in the magnetic formula $I = \kappa H$). Thus we have at once

$$K = 1 + 4\pi n\alpha \qquad \text{......(7).}$$

This formula gives the product of the number of doublets and their polarizability in terms of the measured ratio K. If n is assumed known then this formula gives the electric moment per unit field strength of the doublets in the medium. If we consider that, as in a gas for instance, we can take the particles to be molecules, then n at N.T.P. is Avogadro's Number. If, further, we assume the molecule to behave like a conducting sphere then we can easily show that

$$\alpha = a^3,$$

where a is the radius of the sphere. In unit volume of the dielectric the space occupied by n molecules is $4\pi na^3/3$. Calling this θ we have

$$K = 1 + 3\theta \qquad \text{......(8),}$$

which is the formula given by Jeans's presentation of Mosotti's treatment(8), although he only obtains it as an approximation, since he puts $\sigma = n\alpha E_A$ instead of the correct $\sigma = n\alpha E$.

When we turn to Clausius's expression(6)

$$\frac{K-1}{K+2} = \frac{4\pi n\alpha}{3} \qquad \text{......(9)}$$

we find that this is obtained in the presentation given by Lorentz(7), Debye(8) and others by considering the force among molecules in a spherical volume which has been removed from a cavity in the material. Now this proof is a curious mixture of macroscopic treatment (force in a cavity) and microscopic treatment (force between molecules).

On the macroscopic theory the force is uniform everywhere in the dielectric, and the polarization, being proportional to it, is uniform also. This involves a discontinuity in the value of the force at the boundary of the dielectric. On the microscopic theory there is no discontinuity in the force as the boundary is passed, but the force varies with position and also, possibly, with time. The polarization, if assumed proportional to the force, cannot be uniform everywhere: Lorentz, who has used this treatment with considerable diffidence (9), has shown that at the centre of a spherical distribution of uniformly polarized doublets "having a regular cubical arrangement" the force is zero, and Debye assumes this to be the case as regards liquids and gases also. This result, however, obviously cannot hold near the boundary, where a completely spherical distribution cannot be obtained. This difficulty is overcome in the Clausius-Lorentz treatment by placing the sphere (which, of course, is large compared with molecular dimensions) well inside the dielectric and evading the boundary difficulties by invoking Poisson's macroscopic fictitious-layer theory. This mixture of the two treatments does not seem to be justifiable. The polarizing field in the dielectric on the macroscopic theory is the Internal Electric Field ($E=E_A-4\pi\sigma$, cf. § 6), so that this theory cannot be made use of to show that the polarizing field is really $E_A-4\pi\sigma+4\pi\sigma/3$, as is done in the Clausius-Lorentz proof.

Jeans, who recognizes that the force varies between the molecules, notices that there is a marked difference between the Mosotti formula and that of Clausius. This difficulty he overcomes by recalling his (unnecessary) restriction of the formula to very small values of K. He adds, however, that Pagliani finds that the Mosotti formula agrees better with experiment than that of Clausius. Van Vleck (9) justifies the Clausius-Lorentz expression by pointing out that when n^2 (where n is the refractive index) is substituted for K, it gives a rather better agreement for air at changing pressures than does (7). The substitution of n^2 for K involves extra hypotheses with regard to the effects of rapidly changing forces, so that Pagliani's results must be considered as a more direct experimental test and they favour our formula (7).

No good purpose is therefore served by long discourses on what may happen in the spaces between particles, especially as our formula

$$K=1+4\pi n\alpha$$

holds in all cases where, as we have stated, the medium is ellipsoidal in form and isotropic in nature, and can be considered to consist of doublets whose strength is proportional to the field in which they lie. The only other condition is that the size and distance apart of the doublets shall be very small compared with any dimensions involved in an act of measurement actual or theoretical. This, of course, is true in practice except for the case of gases at very low pressures. The only question that can arise is that of the value to be taken for n, the number of doublets.

The restrictions mentioned by Jeans and Debye would appear to be due to errors in the derivation of their formulae. The error in the Mosotti-Jeans formula

$$\frac{K-1}{K} = 4\pi n\alpha$$

is due to taking $\sigma = n\alpha E_A$ instead of $\sigma = n\alpha E$. This causes an error of a magnitude already mentioned at the end of the last section. The error in the Clausius-Lorentz formula arises through mixing macroscopic and microscopic treatments. For gases, where the value of K is very close to unity, we can write $K+2=3$ and then (9) agrees with (7). However, dipole moments have been calculated for many substances the dielectric constants of which are very far from unity. Writing the Clausius-Lorentz formula as

$$n\alpha = \frac{K-1}{4\pi}\left(\frac{3}{K+2}\right)$$

we see that it differs by the factor in brackets from our formula (7). If we take now one of the worst cases, viz. methyl alcohol, for which K is given as about 35, the factor in brackets becomes approximately 0·08 instead of unity, making a difference of 1200 per cent.

§ 8. PROOF THAT THE INDUCED ELECTROMOTIVE FORCE $\mathrm{E} = \partial/\partial t(\iint \mu H_n \, ds)$

We must now proceed to show that in Neumann's formulation of Faraday's induction experiments, viz.

$$\mathrm{E} = \partial N/\partial t \qquad \ldots\ldots(10),$$

N is the flux of the normal *Induction* enclosed by the circuit or cut by the conductor, and is the sum of elements of the form

(area × normal magnetic field strength × μ),

where μ is the permeability of the medium as defined in § 6; or, in other words, that the μ in $\mu = B/H$ is the same as that in $\mathrm{E} = \partial/\partial t\,(\iint \mu H_n \, ds)$.

This cannot be said to be shown clearly in any text-book and many do not even attempt it.

Now Helmholtz and Lord Kelvin first showed that, given Ampère's laws governing the forces on conductors in a magnetic field and assuming the conservation of energy, Faraday's experimental relation (10) followed necessarily.* Ampère, it must be remembered, performed his celebrated experiments in air, and these deductions apply only to changes in the flux of the normal magnetic *force*. It is easy, however, with care in definition and in the presentation of the theoretical-experiment, to show that in a medium of permeability μ the electromotive force induced is determined by the normal Induction μH_n or B_n.

For this purpose it is easiest to consider a modification of Maxwell's treatment given by Starling in his *Electricity and Magnetism*.

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* Their proofs are not entirely satisfactory. See Maxwell, *A Treatise in Electricity and Magnetism*, 3rd edition, p. 192.

Consider a piece A_1B_1 of a circuit in which a current i is flowing (figure 3) and let H be the Internal Magnetic Force making an angle θ with A_1B_1. For the purpose of utilizing the principle of the conservation of energy in our theoretical-experiment we require the piece A_1B_1 to move a small distance δx under the action of the force acting upon it. Let it do this on conducting rails A_1A_2 and B_1B_2 which are produced parallel to the direction of the force. In order that this may be possible we must have a rectangular slot in the medium whose thickness may be negligible but whose sides must extend to cover $A_1B_1B_2A_2$. Thus we have a slot with sides parallel to the current element and to the direction of the force on it. In such a slot the force has been defined as the Electromagnetic Force or the normal Induction, and its value clearly is

$$H \sin\theta + 4\pi I \sin\theta = (H + 4\pi I) \sin\theta = B \sin\theta.$$

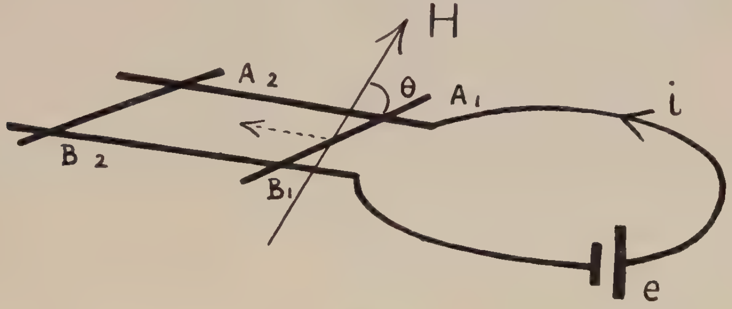

Figure 3.

The rest of the proof follows exactly that given by Starling with the substitution of B for H and need not be repeated here. We conclude therefore that if N is the total normal Induction over the whole circuit

$$\mathrm{E} = \partial/\partial t(N) = \partial/\partial t(\iint B_n ds) = \partial/\partial t(\iint \mu H_n ds) \qquad \ldots\ldots(11).$$

It is clear therefore that the μ in (11) above is the same μ as that in $B = \mu H$. The induced e.m.f. is opposite in direction to the e.m.f. causing the current i if the movement of the conducting piece is such as would take place under the action of the electromagnetic forces themselves. This fact has been the cause of the custom of adding a minus sign and writing $\mathrm{E} = -\partial N/\partial t$, and also, in Neumann's statement of the law, the cause of the emphasis on the *diminution* in the number of tubes of Induction enclosed by the circuit. But the magnitude of the e.m.f. depends only on the *change* in the number of tubes of Induction regardless of whether they increase or decrease. And, further, the minus sign is meaningless when, as happens in almost every case, there is no e.m.f. in the circuit apart from the induced one. This curious custom is a plentiful source of confusion in signs and is used to enable many text-book writers to get their signs correct in Maxwell's equations without considering Fleming's Right-Hand Rule (or its equivalent). The complicated relation in three dimensions between the direction of the induced e.m.f. and the direction and change of the Induction cannot be incorporated in the minus sign alone. It is for this reason that no minus sign has been introduced here: if the direction of the induced e.m.f. is required, Fleming's Right-Hand Rule (or its equivalent) must be used.

The following definition is therefore suggested in preference to that usually given:—

The law of electromagnetic induction. Whenever the number of tubes of Induction which are enclosed by a circuit is changing, there is an electromotive force acting round the circuit equal to the rate of change of the number of tubes of Induction. The direction of this electromotive force can be obtained from Fleming's Right-Hand Rule, a convenient form of which is the following: look along the tubes of Induction towards the circuit; if the Induction is increasing, the induced current is anti-clockwise, if decreasing, the induced current is clockwise.

§ 9. AN ANCHOR-RING DEFINITION OF μ ANALOGOUS TO THE CONDENSER DEFINITION OF K

We can now proceed to make use of the result of the last section to re-define μ in a manner analogous to the definition of K and equally closely dependent on actual experimental procedure. For the purpose of this experiment we choose an anchor-ring closely and completely wrapped with a large number of turns of wire (the "primary"): round the primary are wrapped a number of turns of wire connected to a ballistic galvanometer (the "secondary"). The ratio of the diameter of the anchor-ring to that of the turns of wire, and the ratio of the diameter of the turns of wire to that of the actual wire itself should be as large as possible. The greater these ratios the more closely does the actual experiment comply with the assumptions made in the treatment given.

We first suppose the anchor-ring removed so that the coils enclose only air (more strictly a vacuum) and we cause a current i to be established in the primary. This causes a throw of the galvanometer θ_0. The space between the coils is then filled with the material under investigation and the current i again established with a resulting throw of θ. We must now show that

$$\mu = \theta/\theta_0,$$

where the permeability of a vacuum is taken as unity. If the above conditions of construction are fulfilled, the magnetic field H_A inside is very nearly uniform and equal to $4\pi ni$, where n = number of turns of primary per cm.

The flux associated with the secondary is

$$mAH_A,$$

where m = number of turns in the secondary,
A = area of cross-section of primary.

The e.m.f. produced in the secondary is therefore

$$E_0 = \partial/\partial t(mAH_A).$$

When the material of permeability μ is substituted, the Internal Magnetic Field H is

$$H = H_A - F_1.$$

But there are no free poles, so that $F_1 = 0$. Hence in this special case we have

$$H = H_A$$

and the e.m.f. produced is given by

$$\mathrm{E} = \partial/\partial t(mA\mu H) = \partial/\partial t(mA\mu H_A).$$

The quantities of electricity discharged through the ballistic galvanometer are

$$Q_0 = \int \frac{\mathrm{E}_0}{R}\,dt = \frac{mAH_A}{R}$$

and

$$Q = \int \frac{\mathrm{E}}{R}\,dt = \frac{mAH_A}{R}\cdot\mu;$$

so that we have finally the result desired, viz.

$$\mu = Q/Q_0 = \theta/\theta_0.$$

Thus we are entitled to define the permeability as follows:

"The permeability of a material can be measured as the ratio of the quantities of electricity discharged through the secondary circuit of an anchor-ring when a fixed current is established in the primary, where in the first case the material, and in the second case a vacuum, is enclosed by the primary circuit."

This is exactly analogous to the definition of the dielectric constant:

"The dielectric constant of a material can be measured as the ratio of the quantities of electricity discharged by a condenser which has been charged to a fixed potential, where in the first case the material, and in the second case a vacuum, is enclosed between the plates."

It is therefore possible to define both K and μ in ways which are based directly on their experimental determination and show that they are equivalent measures of the polarizability, electric and magnetic, of different media (§ 6). The analogous positions which they occupy in Maxwell's electromagnetic theory are therefore justified.

§ 10. A NOTE ON DISPLACEMENT CURRENTS

Some writers have adopted the following method for introducing μ: because we cannot perform the condenser experiment in magnetism let us assume the formula $m_1m_2/\mu d^2$ so that it is analogous to e_1e_2/Kd^2. (Those writers who define μ as B/H usually omit to show that this leads to $m_1m_2/\mu d^2$.) Now one of the main objects of this paper has been to demonstrate that the treatment of the effects of electric and magnetic polarization is exactly similar in the macroscopic theory. We could, therefore, with equal cogency say with regard to the anchor-ring experiment: because we cannot perform the anchor-ring experiment in electrostatics let us assume

$$\mathrm{H} = \partial/\partial t\left(\iint KE_n\,ds\right) \qquad \ldots\ldots(12),$$

where H is the magnetomotive force, so that it is analogous to

$$E = \partial/\partial t \left(\iint \mu H_n \, ds\right) \quad \text{(equation 11).}$$

The confirmatory experiments of Eichenwald can then be quoted (10). Application of Stokes's Theorem immediately produces

$$\frac{K}{c}\dot{E} = \text{curl } H, \quad -\frac{\mu}{c}\dot{H} = \text{curl } E.$$

In this way we can obtain Maxwell's equations for an insulator without reference to displacement currents which, while plausible in dielectrics, become difficult to interpret in free space. Another advantage of this treatment is that since we already accept the fact that there is still an e.m.f. induced in the secondary of our anchor-ring when nothing but a vacuum surrounds the coils, there is no reason for surprise that the magnetomotive force still has a value when the dielectric substance gives place to a vacuum.

The beautiful symmetry of Maxwell's equations for a non-conductor are a reflexion of the symmetrical treatment adopted, in particular the concept of a magnetic pole analogous to the electric charge. The fundamental difference between electricity and magnetism, viz. that charges of opposite sign can be separated, whereas magnetic poles cannot, justifies us in considering that all magnetic force is produced by the movement of electric charge rather than that all electric force is the result of the movement of magnets. The separation of electric charge makes electric currents possible, whereas magnetic currents are not, and as soon as we take account of these currents the symmetry of Maxwell's equations disappears.

§ 11. EFFECT OF A MEDIUM ON THE FORCES BETWEEN TWO ELECTRIC POINT CHARGES AND BETWEEN TWO MAGNETIC POINT POLES

The formula $e_1 e_2 / K d^2$ has not been shown to hold in general. To show that it does hold we must first consider the force *per unit charge* due to a charge e_1 surrounded by a medium of dielectric constant K. Consider the positive charge e_1 in figure 4. The medium will be polarized radially (by symmetry) and this will result in a spherical surface-distribution of negative charge in the neighbourhood of e_1, at a distance d_0 (say) and a positive surface-distribution on the boundary at infinity which will produce no force inside. Hence for the Internal Electric Force per unit charge at the centre of a rod-like cavity at a distance d we have

$$E = E_A - F_1.$$

The negative charge, since it is uniform, will act as though concentrated at the centre. If we now call β the Electric Susceptibility (so that $\sigma = \beta E$ corresponds to $I = \kappa H$) the surface density will be

$$\sigma = \beta \times \text{Internal Electric Force at distance } d_0$$

$$= \beta \times \quad ,, \qquad ,, \qquad ,, \qquad ,, \quad d \times (d^2/d_0^2)$$

$$= \frac{\beta E d^2}{d_0^2}.$$

$\therefore$ Total charge $= 4\pi d_0^2 \times \beta E d^2 / d_0^2 = 4\pi \beta E d^2$.

So in the rod-shaped cavity we have

$$\begin{aligned} E &= E_A - 4\pi\beta E d^2/d_0{}^2 \\ &= \frac{E_A}{1+4\pi\beta} = \frac{E_A}{K} \qquad \text{......(13)} \\ &= \frac{e_1}{Kd^2}. \end{aligned}$$

This, therefore, is the force *per* unit charge: if we now place a charge e_2 at the distance d, the physical state of the dielectric is no longer the same since e_2 itself produces polarization which will cause a surface charge to appear on the walls as well as the ends of the cavity. We can, however, treat this by means of the principle of superposition and say that the total polarization is the sum of that due to e_1 acting alone plus that due to e_2 acting alone since it is everywhere proportional to the electric force. The charge e_2 acting alone in the centre of a symmetrical cavity will produce a symmetrical distribution of negative charge and this will produce no resultant force at the centre. The polarization in the neighbourhood of e_1 will be negligible compared with that due to e_1 itself *provided that all distances are large compared with the molecular structure*, an assumption on which, of course, the whole classical treatment already rests. Thus the presence of e_2 does not alter the polarization in a manner which would alter the force in a rod-like cavity, and we can therefore write

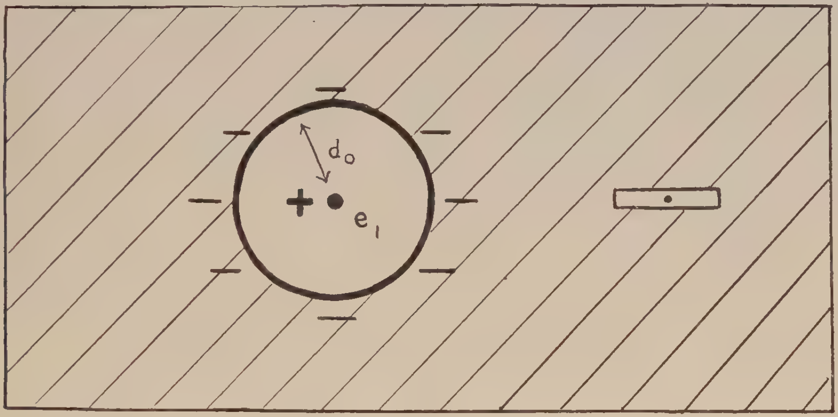

Figure 4.

$$e_1 e_2/Kd^2 \qquad \text{......(14)}$$

for the force between two point charges e_1 and e_2 separated by a distance d large compared with the dimensions of the molecular structure in a medium of dielectric constant K.

Clearly the treatment above applies equally to the case of two magnetic point poles separated by a distance d in a medium of permeability μ, and so we have also

$$m_1 m_2/\mu d^2 \qquad \text{......(15).}$$

Continuing in a similar manner we may investigate the forces between magnetic poles and current elements and between current elements themselves.

In each case we can show by the principle of superposition of effects that the polarization produced by the object on which the force is measured does not affect the value of the force if it is at the centre of a symmetrical cavity.

§ 12. EFFECT OF A MEDIUM ON THE FORCES BETWEEN MAGNETIC POLES AND CURRENT ELEMENTS AND BETWEEN TWO CURRENT ELEMENTS

Pole on current element : Consider first the case of a current element at right angles to the lines of force of a magnetic point pole m (figure 5) and at a distance d from it.

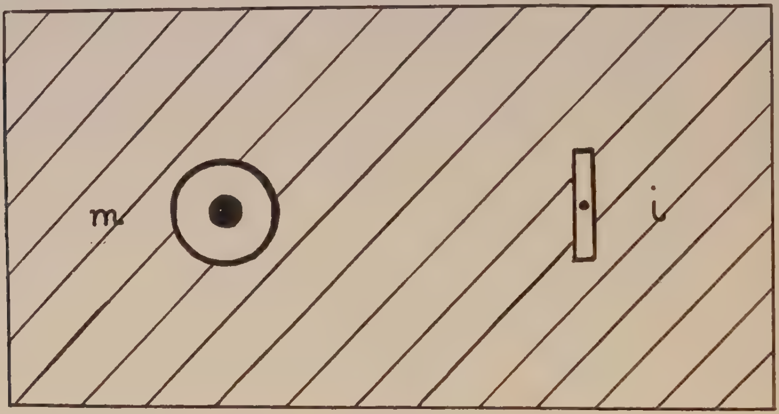

Figure 5.

Induction in disc-shaped cavity $B = H_A - F_1 + F_2$. Now $H_A - F_1 = H = H_A/\mu$ from comparison with (14) and $F_2 = 4\pi I$.

Hence
$$\begin{aligned} B &= H_A/\mu + 4\pi\kappa H_A/\mu \\ &= H_A(1+4\pi\kappa)/\mu \\ &= H_A \text{ (the same as in air).} \end{aligned}$$

Current element on pole :

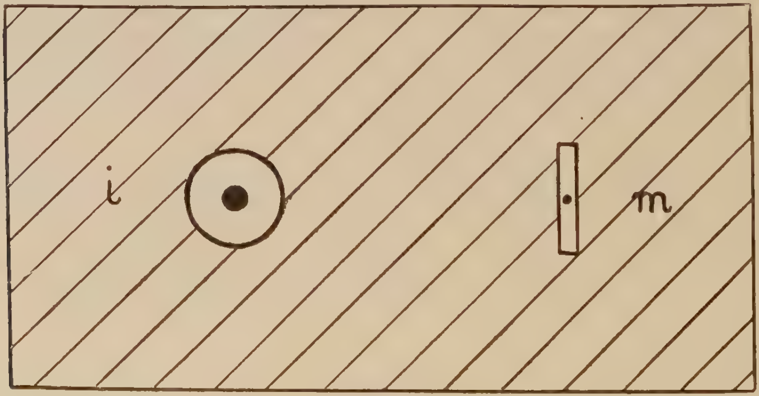

Figure 6.

Force in rod-shaped cavity $H=H_A-F_1+F_2$.

In this case there are no free poles round i, so that $F_1=0$. $F_2=0$ also, whence

$$H=H_A \qquad \ldots\ldots(16),$$

which is the same as in air.

Two current elements:

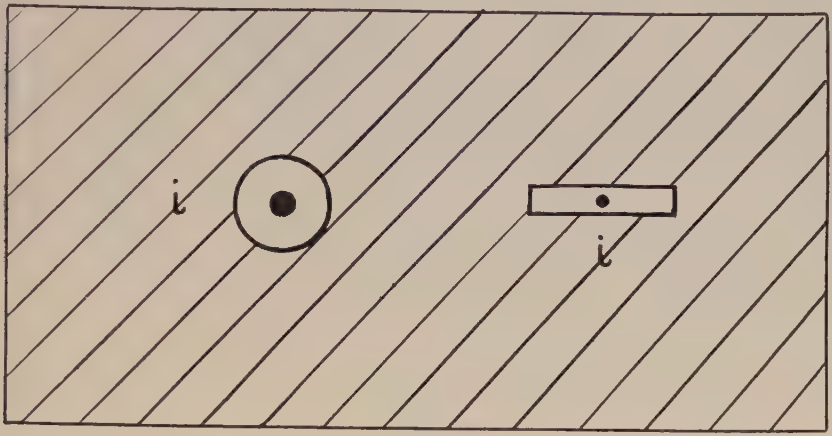

Figure 7.

Induction in disc-shaped cavity $B=H_A-F_1+F_2$. $F_1=0$ as before : $F_2=4\pi I$, whence

$$\begin{aligned} B &= H_A+4\pi I \\ &= H+4\pi I \text{ since from (16) } H=H_A \\ &= \mu H \\ &= \mu H_A \\ &= \mu \text{ times the force in air.} \end{aligned}$$

Thus we see that for the hypothetical point pole and current element the effect of the medium is to *decrease* the force between two poles, *increase* the force between line elements of current, and leave the force between poles and current elements unchanged.

§ 13. EQUIVALENCE OF A CURRENT CIRCUIT AND A MAGNETIC SHELL IN ALL MEDIA

The confusion mentioned in § 1 may now be discussed. This arises through forgetting that the forces due to magnets or current circuits immersed in a polarizable medium depend on the shape of the cavity in which they are placed, and further, that a current circuit can be put in two different cavities whereas a magnetic shell cannot. To see this clearly, consider figures 8, 9 and 10. The current circuit can be put in the medium either by removing a cylindrical channel to take just the wire itself (figure 8) or by inserting it as a whole in a disc-shaped cavity (figure 9). The latter cavity is suitable for the equivalent magnetic shell (figure 10) whereas the former is not. It is easy to see that as long as we treat the circuit and magnetic shell in a similar manner they will be equivalent in the medium just as they are in a vacuum.

Consider the Internal Magnetic Force at any point P in each case : in figure 8 there is no surface distribution of free pole, so that $F_1 = 0$. Hence $H = H_A$ and the field is the same as *in vacuo*. In figures 9 and 10 a surface distribution of pole of amount I appears, equivalent to a row of doublets acting in the opposite direction to the fields of the current and the shell. This produces a force F_1 at P, so that we have now $H = H_A - F_1$. We can easily show that this becomes H_A/μ. Let ϕ be the strength of the shell, i.e. the magnetic pole per unit surface. I is the magnetic pole per unit surface of the cavity, so that at P the potential, which is

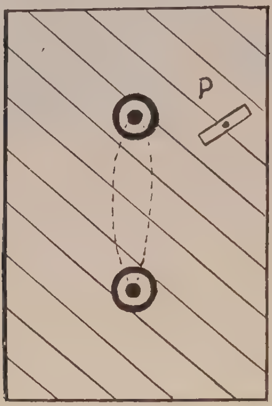

Figure 8.

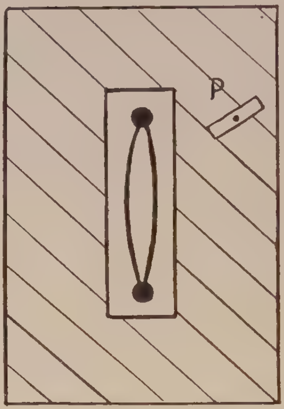

Figure 9.

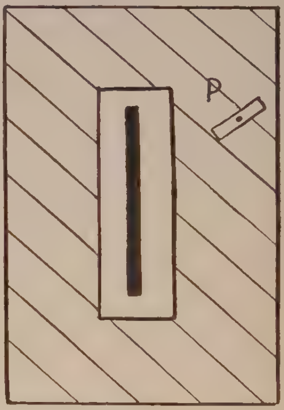

Figure 10.

$$\Omega = \omega\phi \text{ in a vacuum}$$

(where ω is the solid angle subtended by the shell or circuit at P), becomes

$$\begin{aligned} \Omega &= \omega(\phi - I) \text{ in the medium} \\ &= \omega\left(\phi - \frac{4\pi\phi\kappa}{\mu}\right) \\ &= \omega\phi\,(\mu - 4\pi\kappa)/\mu \\ &= \frac{\omega\phi}{\mu}. \end{aligned}$$

Thus the potential, and consequently the force, is reduced in the ratio $\mu : 1$. This clearly applies also to the current circuit in figure 9.

The difference between the cases illustrated in figure 8 and figure 10 has led many authors of well-known text-books to be at pains to explain that a current circuit and a magnetic shell which are equivalent in air are no longer so in a medium of permeability μ. In fact, it is maintained that the magnetic shell of strength i in air must be of strength μi in the medium. We have seen, however, that a current circuit and a magnetic shell which are equivalent in a vacuum are equivalent in any medium provided that they occupy similar cavities. The force they produce is, in each case, diminished in the ratio $\mu:1$. A current circuit in which the wire is entirely surrounded by a medium, as is normally the case in gases and liquids, exerts the same magnetic force as in a vacuum.

If we consider the force on a current element at P, i.e. the force in a disc-shaped cavity whose axis is parallel to the magnetization, we find:

figure 8. $B=\mu H$ and $H=H_A$; $\therefore B=\mu H_A$, or μ times as great as in a vacuum.

figures 9 and 10. $B=\mu H$ and $H=H_A/\mu$; $\therefore B=H_A$, or the same as in a vacuum.

Thus when a current circuit is entirely surrounded by the medium, as in figure 8, the Internal Magnetic Force is unaltered but the Internal Electromagnetic Force or Induction is μ times as great.

If the circuit is in a cavity as in figure 9, the Internal Electromagnetic Force is unaltered and the Internal Magnetic Force is μ times less.

No difference arises, however, in the cases illustrated in figures 9 and 10, whether we consider the respective magnetic or the electromagnetic forces. We can conclude therefore that a current circuit and a magnetic shell are wholly equivalent in all media, provided they occupy similar cavities.

This equivalence also applies to Ampère's molecular currents, since we do not suppose that the medium penetrates the ultimate magnetic particle. Thus the contradiction mentioned in § 1 is resolved.

§ 14. EFFECT OF THE MEDIUM ON THE FORCES BETWEEN ACTUAL MAGNETS AND COMPLETE CURRENT CIRCUITS

The dependence of the field on the type of cavity in which the circuit is placed makes it important to inquire what happens in the case of actual magnets and current circuits. In these cases it will be necessary to consider as well the effect of cavities in which the measurements are made, since the assumptions made in choosing our rod-shaped and disc-shaped cavities (suitable only for point-poles) will not in general hold; the direction and intensity of the magnetization may not, for instance, be constant over the region occupied by the cavity. Such cases will therefore require special treatment.

One case of importance can be dealt with: that of the current balance which is used for defining the unit of current. Here the current coils are situated as in figure 8, i.e. surrounded entirely by air or partly by other materials such as marble. The force between the coils is therefore greater than would be the case in a vacuum. The very small value of the permeability of air makes a correction unnecessary.

§ 15. THE FIVE ELECTROMAGNETIC RELATIONS

We can now re-state in rather more precise form than usual the five electromagnetic relations:

(1) The magnetic potential at a point produced by a current flowing in a closed linear conductor is equal to the strength of the current i multiplied by the

solid angle ω which the conductor subtends at that point. This is independent of the medium provided that it completely surrounds and penetrates the circuit:

$$\Omega = i\omega \qquad \text{......(17).}$$

(2) The contribution to the magnetic field produced by a current i flowing in an elementary length ds of a conductor is given by

$$dH = \frac{i\,ds\sin\theta}{r^2} \qquad \text{......(18),}$$

where r is the distance from ds to the point considered and θ is the angle between r and ds. The direction of dH is perpendicular to the plane containing r and ds, and its sense can be obtained by imagining the conductor grasped in the right hand with the thumb pointing in the direction of the current, the fingers then pointing in the direction of the field. This formula is independent of the medium.

(3) The work done in carrying a unit magnetic pole once round any closed path is 4π times the algebraic sum of the currents flowing through the closed path and is independent of the medium :—

$$\mathrm{W} = 4\pi i \qquad \text{......(19).}$$

(4) The mechanical force on an element of conductor ds carrying a current i is given by

$$dF = i\,ds B\sin\theta \qquad \text{......(20),}$$

where B is the magnetic Induction in the region occupied by ds and θ is the angle between B and ds. The sense of dF is given by Fleming's Left-Hand Rule.

(5) The law of electromagnetic induction

$$\mathrm{E} = \partial/\partial t\left(\iint B_n\,ds\right) \qquad \text{......(21).}$$

This has already been defined at the end of § 8.

§ 16. FROM THEIR DEFINITION K AND μ ARE DIMENSIONLESS

From the considerations advanced in § 9 it is clear that both K and μ are ratios, and moreover are ratios of numbers obtained by *the same measuring operation.* Consequently when the numeral and the symbol or symbols representing the measuring operation (e.g. cm.) in one case (the medium) are divided by the numeral and symbol representing the measuring operation in the other case (the vacuum) we are left with pure numbers, and this is what both K and μ must be.

The confusion which has always vitiated the theory of dimensions arises as follows :—According to Maxwell's electromagnetic theory the velocity of electromagnetic waves in a medium of dielectric constant K and permeability μ is $c/\sqrt{K\mu}$, where c is the number of electrostatic units in one electromagnetic unit of current. This makes c appear to be a pure number, but once again, if we examine the *operations* by which c is determined, we see at once that it is not a pure number.

An easy case to consider is the comparison of the capacities of the same condenser on the two systems, which gives the value of c^2 To find the capacity of a condenser on the E.S. system we require only a ruler, i.e. we need only measure space-like intervals, whereas to find the capacity on the E.M. system we must measure both space-like and time-like intervals. It follows that on division the symbols representing the measuring operations cannot cancel out and we are left with c^2 cm^2 per sec^2—not just a pure number.

Consequently it is c which has the dimensions of a velocity and not $1/\sqrt{K\mu}$.

It follows from the relation $B=\mu H$ with μ a pure number that the Induction and the Internal Magnetic Force have the same dimensions. The question has arisen as to whether they represent " quantities of a different nature ": the Paris Committee of the I.E.C. decided by a majority that they did. It is hoped, in a future communication, to deal with a new presentation of the theory of dimensions where this problem will be discussed in detail. Here, therefore, we shall only make a short comment.

Physically the Induction is a measure of the interaction of a magnetic field and currents already flowing, or induced in, a conductor. We have seen that this interaction could have been measured as the Electromagnetic Force, but it has been defined in the classical treatment of Maxwell and Kelvin as the force on a *magnetic pole* in an arbitrary cavity rather than as the force on a *current element*, and its action on the element has therefore been forced to be perpendicular to its direction. We have seen, further, that the Electromagnetic Force is not only always perpendicular to the Magnetic Force, but is only numerically equal when at its maximum value in a vacuum.

It would therefore be quite reasonable to have a unit with a different name to signify that the force is measured per unit current per unit length of conductor rather than per unit magnetic pole, in a somewhat similar way to the distinction made between the force per unit of mass (dyne) and the force per unit of pole (gauss). All are forces, and all could, in theory, be measured with a spring balance: the different names of the units merely indicate the nature of the test-particle.

On the classical theory, however, where both Magnetic Force and Induction are in the same direction and defined as forces measured on magnetic poles, such a distinction in the unit does not seem equally justified.

§ 17. SUMMARY AND CONCLUSION

Prof. H. Abraham, secretary of the Committee of the International Electrical Congress, has maintained that there are at least six different definitions of the permeability which result in six vectors which he calls B_1, B_2, B_3, B_4, B_5 and B_6, and that it is not possible to show that they are either the same or different. Now Abraham's B_1 is the B in our formula (20), his B_2 is the B in our formula (21), and his B_3 is concerned with the μ in our formula (15) and the discussion of § 14. We have, therefore, shown that Abraham's first three (B_1, B_2 and B_3) are identical. His B_4, B_5 and B_6 refer to magneto-optics and are not of the same fundamental importance.

In discussing the use of theoretical-experiments and the meaning of " force on a pole " and " force on a current element " we have been led inevitably to the rod-shaped and disc-shaped cavities of Kelvin and Maxwell, which do not therefore have to be arbitrarily introduced as they are in the classical theory.

A formula resembling that usually known as the Clausius-Mosotti formula has been simply derived from the macroscopic theory without any irrelevant discussion of forces existing between, or within, molecules; and the causes of what would appear to be errors in the derivation of the Mosotti-Jeans and Clausius-Lorentz formulae have been discussed.

In order to emphasize the fact already demonstrated that K and μ are analogous measures of the polarization of a medium, an anchor-ring definition of μ has been given which is similar to the condenser definition of K and equally closely dependent on actual experimental procedure. This leads to a derivation of Maxwell's equations without invoking the unsatisfactory hypothesis of displacement currents in free space.

A general proof has then been given of the validity of the formulae $\frac{e_1e_2}{Kd^2}$ and $\frac{m_1m_2}{\mu d^2}$ and the effect of the medium on the forces between magnetic poles and current elements and between current elements determined. The equivalence of a current circuit and a magnetic shell in all media has been established and the confusion arising from the two possible cavities for a circuit has been stressed. It is concluded that the force between the coils of a current balance, if not in a vacuum, should be multiplied by the permeability of the medium surrounding the coils.

The five electromagnetic relations due to Ampère, Faraday, Laplace and Neumann have been re-stated with the effect of the medium explicitly mentioned. Finally, the dimensionless nature of K and μ has been emphasized and the value of treating the Induction and the Internal Magnetic Force as quantities of a different nature discussed.

In conclusion, we cannot do better than visualize the deliberations of two committees; one which took place in 1932 and one which probably took place 2500 years earlier. In 1932 the problem before the Paris Committee—a joint committee appointed by the International Electrotechnical Commission and the International Union of Pure and Applied Physics—was as follows: given the intuitive assumption of a great Gothic field-of-force myth, viz. that at every point in infinite space there was one unique magnetic force acting—how could it come about that in a polarizable medium there were two magnetic forces acting at one and the same point? Were they really different? By a majority, the committee decided that they were. Two thousand five hundred years before, the problem was this: given the intuitive assumption of the Classical number-idea, viz. that numbers represented tangible and visibly-limited units and consequently could only be positive and whole—how could it be explained that if one side of a square was a whole number of units then the diagonal could not be? The Pythagoreans decided to call it a mystery and introduce it into their religion. The Paris Committee decided to call it an Oersted and introduce it into physics.

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DISCUSSION*

Professor E. W. Marchant. The method of teaching the fundamental principles of electric and magnetic theory is a matter of great importance to electrical engineers. During the last few years I have used, with my second-year students, the M.K.S. system, and have found it quite convenient, if it is used consistently. As a rule these students are taught the C.G.S. system of units, and it takes some time for them to become familiar with the M.K.S. system, which gives the practical electrical engineering units as absolute units. It is important to make clear the distinction between magnetic force and magnetic induction and between electric force and electric displacement. I think it is a great advantage to take these two sets of quantities as analogous and to regard both the permeability of space and its permittivity as qualities which are dimensional. In the M.K.S. system the numerical value of neither of these two quantities is unity, and there is, therefore, a distinction between the quantities which emphasizes the fact that they are not alike. In this system of units the definition of the magnetic and electric quantities is quite consistent. Magnetic force is defined as the force on a unit pole placed in the space. In a permeable substance, like iron, the magnetic force must be defined as the force on a unit pole placed in a long narrow tunnel. In the same way electric force can be defined in terms of the mechanical force exerted on a unit electric charge or unit of quantity. Magnetic induction is defined in terms of the space permeability of the medium in which the magnetic force producing it exists, and is measured by the e.m.f. induced in a circuit, when the flux through it is changed, the flux being the integral of induction times area. It may also be measured by the force exerted on a current-carrying conductor. Electric flux density is, of course, measured in terms of quantity of electricity per unit area; the displacement current is the quantity per second flowing across the dielectric of a substance which is under an electric stress.

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* Partly on matters cognate to, but wider than, Dr. Brown's subject.

In a book which has recently been published by Messrs. Methuen I have set out the implications of the M.K.S. system and the necessity for differentiating between magnetic force and magnetic induction. I hope that the teaching of physics will be carried out along these lines, so that there may be less confusion than exists at present as to the nature of these quantities. It would be a great advantage if teachers of physics could see their way to using the system of units which is employed by practical electrical engineers.

Mr. C. R. Cosens. In teaching electromagnetism to undergraduate students, more particularly to those whose interests lie primarily in the application of physics to engineering, there is a very real danger that his familiarity with the subject may lead the teacher to forget his own early difficulties ; the conceptions involved (e.g. *magnetic flux*) are quite foreign to the learner's experience, and are much more difficult for him to visualise than, say, "pressure intensity"; for among his earliest memories he has a working, if not strictly defined, conception of the ideas *pound* and *inch*, so that *pounds per square inch* conveys to him at least some sort of physical meaning. The difficulty is unnecessarily increased by the redundancy of units of measurement, of which the student is expected to memorise the names and definitions—an exercise of the same value as that of memorising irregular Greek verbs, but not calculated to accelerate his progress in the early stages. The redundant names appear to be largely by-products of the labours of International Commissions, and are apparently given in order to perpetuate the immortal memory of some illustrious physicist ; if such memory be indeed immortal, this is surely unnecessary, and one may be pardoned for wondering what other object is attained by multiplying these "memorial" names of units.

In elementary magnetism we have five units—*Maxwell*, *Weber*, *Gauss*, *Oersted*, and *Gilbert*—only two are needed ; if flux is to be measured in *Maxwells*, why not flux-density in *Maxwells per square centimetre*, which is self-explanatory, instead of *Gausses* ? The engineer gets on quite well measuring stress-intensity in $lbs./in^2$, without coining a special name for it, such as *Hookes* (and he will probably continue to measure flux in *lines* and induction in $lines/cm^2$, International Conventions notwithstanding). If the M.K.S. system become universally adopted, as so ably advocated by Vigoureux and Webb in their *Electric and Magnetic Measurements*, the unit of flux is the *Weber* or volt-second; then why not call it a *Volt-second* ? If, following Heaviside, we throw the constant 4π into the "permeability of space", the unit of H becomes *ampere-turns per metre*, as Vigoureux points out (l.c. page 3); it is to be hoped that a "memorial" name will not be thrust upon it.

Turning to the controversial question whether B and H are "quantities of the same nature", I had hitherto inclined to the view that no one has any right to state, as *proved fact*, either a positive or negative conclusion, unless he claim to be the recipient of Divine revelation; but a perusal of Dr. Burniston Brown's paper leads me to suspect that the answer to this question depends on the exact details of the precise definitions of B and H which are adopted. We are told that Professor Abraham distinguishes six vectors, differently defined, and all called B; all represented by the same number (I assume) in a given case, but can it be proved that all of these vectors are "of the same nature" as one another (and that phrase also needs definition !), let alone that any of them is "of the same nature" as any H ? (*Any* H, since it is almost certain that there are several definitions of H.)

I suggest that some of the difficulty with B and H is that we use two symbols to represent at least three fundamentally distinct conceptions. Instead of using "cavity" processes, there is an alternative method of defining B and H by theoretical-experiments more closely allied to the facts we really do know about them, namely, that the time-differential of flux-turns is volts, and that the line-integral of H is ampere-turns. In this treatment we defer consideration of iron or magnetic material until further notice, except for one fictitious "unit pole" which we attach to a brass spring-balance by which the force in dynes upon it is measured, and we call this force F (*not* H).

We then construct a hollow firebrick anchor-ring, large enough for an observer with the unit pole to get inside, of axial length l, measured through the centres of the circular cross-sections, and uniformly wound with T_1 turns of wire. We connect the winding to a battery through a carbon rheostat and blank moving-coil ammeter—*blank* because we have not yet defined the ampere. On closing the circuit and screwing up the rheostat, the observer inside reports from time to time the force F on the unit pole, and we calibrate our ammeter in such a way that its reading is equal to $10Fl/4\pi T_1$, and we have thus defined the ampere. (It is, of course, necessary to show that if we repeat the experiment with a different-sized ring, etc., we get the same calibration; this is not a theoretical assumption, but an experimental fact.) Having thus defined the ampere, we can determine F without the assistance of the observer inside, by means of the equation

$$F=4\pi IT_1/10l \qquad \ldots\ldots(\alpha).$$

We next provide a coil of area A, having T_2 turns, and place it with its plane at right angles to the circular axis of the anchor ring and coaxial therewith, and with the aid of the observer inside we calibrate a voltmeter in terms of the time-differential of F, so that the equation connecting them is finally

$$F=(10^8/AT_2)\int E\,dt \qquad \ldots\ldots(\beta),$$

and we can determine F from the readings E of the voltmeter and a clock.

There is as yet no question but that F in equations (α) and (β) is the same thing; it may be called magnetic force, measured in dynes per unit pole.

Having rescued our observer from within the anchor ring, we fill it with molten cast iron and let it solidify; our observer then reports that the unit pole accidentally became detached from the spring-balance and is now embedded in the interior of the cast iron, and he asks "what is now the force F on it ?" He suggests that we drill a "small" hole in the iron, insert another unit pole, and measure F directly, but we suspect that we should not then be measuring F actually *inside* the iron, but only in air in a hole in the iron, which may be something quite different—a suspicion strengthened by the observation that the value so found depends on the shape of the hole !

It is then suggested that the labour of drilling a hole is unnecessary—we can calculate F from equation (α); we do so, and attempt to confirm our result by calculating from equation (β), to find that the results from these equations are totally different from one another. We therefore, to distinguish these results, give the name H to that calculated from (α), and B to that calculated from (β). Thus H is now, and for the first time, defined by the equation $H = 4\pi I T_1/10l$, and similarly B is defined by $B = (10^8/AT_2)\int E\,\delta t$.

Note that we use F (magnet*ic* force, dynes per unit pole) to define I and E in the absence of iron; in the presence of iron we use I and E so defined to determine H (magnet*ising* force, ampere-turns per unit length) and B (volt-second per unit area).

If we then ask which of these two, B or H, is the *true* value of F, the answer can only be—"I do not know; possibly neither". We are not in a position to state categorically whether either B or H is "of the same physical nature" as F, using the definitions of F, B, H given above; we can only say that this seems possible but unlikely. But, meanwhile, B and H are what we need for practical purposes, the theoretical force F on a non-existent "unit pole" is of no real use; F and the unit pole, having served the purposes for which they were, in imagination, created—the calibration of ammeter and voltmeter—may now be discarded.

(If my reading of Vigoreux and Webb's definitions be correct, it follows that F, the force (in "Newtons") on a unit pole in air, will be numerically equal to neither B nor H in their proposed M.K.S. system.)

I then suggest that "permeability" $\mu = B\ H$ can be dispensed with, as may "reluctance" $\mathfrak{R} = (\text{flux}) \div (\text{magnetomotive force})$, on the grounds that the useful efficiency of such quantities is inversely proportional to the square of their percentage variation from the mean in a given specimen of material. In practice we seldom, if ever, find it convenient to use the resistance $R = V/I$ of a conductor, such as a copper-oxide rectifier, which "does not obey Ohm's Law" (i.e. in which R is not a constant); we prefer to work directly from the graph or equation relating I and V without the intervention of their ratio; similarly in ferromagnetic problems the graph connecting B and H is far more rapidly used than one involving μ.

Susceptibility $\kappa = (\mu - 1)/4\pi$ is apparently essential to the atomic physicist, but there seems no case for the retention of both μ and κ. Could he not with advantage decide which he prefers to retain, and abolish the other ?

I submit that the application of William of Occam's razor, *Entia non sunt multiplicanda praeter necessitatem*, is long overdue in this branch of physics;

it would greatly accelerate the learner's progress without severely handicapping the skilled research-worker. Such "rationalisation" by relentless "scrapping" of redundant tools is as vital in theoretical science as in workshop technology; reform could be imperceptibly yet effectively obtained if examiners would refrain from setting questions such as "define reluctance" in examination papers; they might reasonably expect a candidate to understand a graphical or tabular statement such as $B=10{,}000$; $H=8$: but any other quantities required for the solution of a problem should be given with an *ad hoc* definition.

Mr. W. E. BENHAM (communicated). When I received the proof of Dr. Brown's paper my first feeling was one of slight disappointment at finding that he had still had recourse to the use of cavities. This feeling was, however, displaced by one of admiration for the way in which the subject matter was handled, both in the paper and before the Society. His justification for the selection of rod-shaped and disc-shaped cavities seems to rest mainly upon the freedom to move which must be allowed to exploring charges or poles or current elements, whereas I had hoped he was going to show us how to dispense with cavities altogether. Indeed, the extra complication which might be considered to accompany any treatment of so fundamental a character as to render recourse to cavities unnecessary would seem to me to be well worth while, particularly if it could be applied equally to a collection of magnetic or electric doublets with mere change of symbols. Such a treatment I believe to be entirely within the bounds of possibility. I would say more—that the material for such treatment has already been given—by Langevin in 1905 for paramagnetic gases and by Debye in 1912 for the electrical counterpart in which *permanent* electric doublets were introduced as an essential part of the theory of dielectrics. Langevin's theory forms the basis of more modern developments, which only allow certain values for the orientation of the doublets; but this aspect need not be considered in an elementary presentation, which might proceed as follows.

Let μ be the molecular magnetic (or electric) moment, and let this value be the same for all molecules. The energy of a molecule when it makes an angle θ with the "lines of force" of the resultant field H (or E) (which is Brown's internal force) is, for the magnetic case,

$$\int_0^\theta \mu H \sin\theta \, d\theta = \mu H(1-\cos\theta) \qquad \ldots\ldots(\mathrm{A}).$$

The field H would orient the molecules parallel to itself, but this tendency may be regarded as resisted by thermal agitation. If T be the temperature of the (gas, liquid or solid) in degrees Kelvin, then the number of molecules which make an angle with H lying between θ and $d\theta$ is, assuming a Boltzmann distribution,

$$dN = CNe^{-\mu H(1-\cos\theta)k/T} \sin\theta \, d\theta \qquad \ldots\ldots(\mathrm{B}),$$

where we have taken a unit sphere and a double cone of angle $d\theta$. The constant

C is determined, as usual, by integration for all θ, giving $2C=\alpha \sinh \alpha$, where $\alpha=\mu H/kT$. Now if I is the intensity of magnetization,

$$dI=\mu \cos\theta \, dN=\mu NC \cos\theta \sin\theta e^{\alpha\cos\theta} d\theta \qquad \ldots\ldots\text{(C)},$$

where μN will be the intensity I_0 when all the magnetic molecules are aligned with H. Integrating over the unit sphere,

$$I=CI_0\int_0^{\pi}\cos\theta \sin\theta e^{\alpha\cos\theta} d\theta \qquad \ldots\ldots\text{(D)},$$

whence

$$\frac{I}{I_0}=\coth\alpha-\frac{1}{\alpha} \qquad \ldots\ldots\text{(E)},$$

which, for α small, as is usually the case, gives

$$\frac{I}{I_0}=\frac{\alpha}{3}=\frac{\mu H}{3kT} \qquad \ldots\ldots\text{(F)}.$$

The expression for the susceptibility is

$$\kappa=\frac{I}{H}=\frac{\mu I_0}{3kT}=\frac{\mu^2 N}{3kT} \qquad \ldots\ldots\text{(G)},$$

and this will apply also to the "electric susceptibility" if μ is taken as the moment of electric doublets, so that the expression for K, the dielectric constant, is

$$K=1+\frac{4\pi\mu^2 N}{3kT} \qquad \ldots\ldots\text{(H)}.$$

Comparing with Brown's equation (7) we have for the dipole moment

$$n\alpha=\frac{\mu^2 N}{3kT} \qquad \ldots\ldots\text{(I)},$$

where N and n are, of course, the same. It is, however, possible to show, though I will not inflict the treatment here, that unless the doublets which we have supposed to exist are other than parallel to the exciting field, $K=1$. This means that when $I=I_0$, the susceptibility κ differs from zero not at all, whereas it should by (G) be equal to $I_0 H$ or $\mu N H$. The difficulty in accepting the Langevin treatment as it stands is also apparent from the consideration that two angles are, in reality, concerned and not only one. Thus there is the angle, ϕ say, denoting the actual position of the doublet within the unit sphere, as well as the angle θ which the doublet is supposed to make with the field. This difficulty should be pointed out to students and not glossed over. The result of including both angles in a general treatment is to give for the dielectric constant

$$K=1+A\Sigma\mu \sin\theta \qquad \ldots\ldots\text{(J)},$$

where A is a constant at a given temperature and μ, θ have their previous meanings. The summation is taken over all molecules. (J) has the advantage over (I) that the polarizability is now proportional to μ, as would be expected, instead of to μ^2.

The macroscopic theory involves the assumption that the intensity of magnetization is aligned with the exciting field, a conception which is perfectly correct but which tends to create the impression (subconsciously) that all the *individual* doublets are so aligned. This, however, is not true, and it therefore seems to me that, in teaching the young, the microscopic or molecular aspect should be referred to, and, if possible, explained in some detail. It certainly should not, even for teaching purposes, be relegated to the category of "irrelevant discussion", since sooner or later it will be bound to react to the disadvantage of the student, whose tendency would be to mistrust the whole of macroscopic theory immediately he finds out the truth. These remarks are also intended as a warning against the use of cavities, scooped-out spheres, and the like. If it is not possible to derive the Clausius-Lorenz-Lorentz-Hartree correction $4\pi\sigma/3$ without recourse to "conjuring tricks" of this character, then it is far better that the correction should be taken as zero, so that we end up with Mosotti-Jeans or Brown according to whether we take $\sigma = n\alpha E_x$ or $n\alpha E$. Dr. Brown's contention that the latter is correct is new to me and interesting. To the objections which Brown raised to the treatment of Lorentz may be added the objection that, if an integral number of dipoles are removed from the spherical cavity the walls of the latter will be cut by a certain number of residual dipoles. Before the cavity was formed no free charge existed, but as a result of scooping out the cavity there is a resultant charge density on a macroscopic view. It may, however, be pertinent to enquire whence this charge density arises ; there cannot be any resultant charge on that surface whatever which is not compensated by an equal and opposite charge somewhere else. Hence we cannot scoop out the cavity in question without affecting the charge throughout the body of the material. If, as a result of this complication, we suppose that no resultant charge arises, but that the residual doublets cut by the sphere give rise to a resultant field at the centre of the sphere, still further difficulties are introduced. As a result of such considerations I long ago abandoned the $4\pi\sigma/3$ correction as an unfortunate "non sequitur" on which physicists have wasted considerable time.

But the time wasted on this question is probably insignificant in comparison with that expended on "dimensions".

I have come to the conclusion that the cause of the whole trouble is the readiness with which "cows" were equated to "sheep" in the first place. As an illustration consider the evolution of the equation

$$F = ma \qquad \ldots\ldots(\text{K}).$$

Beginning with Newton (we might have gone still further back), "Mutationem motûs *proportionalem* esse vi motrici impressae, et fieri secondum lineam rectam quâ vis illa imprimitur". (The italics are mine.) All Newton said was that change of motion is *proportional* to the force applied, and it was not until years afterwards that an "absolute" unit of force was introduced by Gauss, according to whom [q.v.] "Unit force is that force which, acting on a national standard unit of matter during the unit of time, generates the unity of velocity".

Thus, whereas all Newton hypothesized was

$$F = N\frac{d}{dt}(mv) \qquad \ldots\ldots(\text{L}),$$

where N is some constant, Gauss went a step further and chose the unit of force in such a way that $N=1$. But what has happened to the "dimensions" of N? The extraordinary thing is that just because of a convenient choice of units, due to Gauss, which happens to make N *numerically* equal to unity, Thomson and Tait, to take two of the most thorough and circumspect authorities of the last century, could, after avoiding the pitfall for 26 pages, write in § 259, p. 245, of their famous *Treatise on Natural Philosophy*

$$M\frac{d^2x}{dt^2}=X \qquad \text{......(M)},$$

without a word as to the qualitative equivalence of the related quantities, despite the fact that (previous section, same page) the remark appeared: "For, if we consider the actions of various forces upon the same body for equal times, we evidently have changes of velocity produced which are *proportional to* the forces". (The italics (this time) are *not* mine.) The rate of change of momentum had, indeed, been previously shown to equal mass times acceleration (§ 212), so that (M) is a combination of § 212 and § 223, in which Gauss's unit of force is proposed. It is evident that the "dimensions" of N were considered to disappear with the fixing of its value at unity. But there is no justification whatever for such a view. In consequence, the constant G of gravitation, generally given as

$$G=6{\cdot}664\times10^{-8}\ \text{cm.}^3\ \text{gm.}^{-1}\ \text{sec.}^{-2},$$

is, in reality, *not* of these dimensions, but of these dimensions multiplied by the dimensions of N. Alternatively, writing

$$F=\frac{GMM'}{R^2}=NMa=NM\,a' \qquad \text{......(N)},$$

where a, a' are the accelerations of M, M' respectively, and equating the product of the first two to that of the second two members,

$$F\frac{G}{R^2}=N^2aa'$$

or

$$F=\frac{N^2}{G}\,aa'R^2 \qquad \text{......(O)}.$$

Writing [ ] for "the dimensions of", and L, T, M for "the dimensions of length, time, mass",

$$[F]=\left[\frac{N^2}{G}\right]\frac{L^4}{T^4} \qquad \text{......(P)}.$$

"It is interesting" said Lord Kelvin (2) ". . . to work out in detail the idea of founding the measurement of lengths of mass and force on no other foundation than the measurement of length and time. In doing so we immediately find that the square of an angular velocity is the proper measure of density or mass per unit volume; and that the fourth power of a linear velocity is the proper measure of a force . . ."

Writing, therefore,

$$[F]=\frac{L^4}{T^4} \qquad\qquad \ldots\ldots(Q),$$

we see from (P) that N^2 and G must then have the same dimensions ; hence, if (Q) is true, the density is given by

$$[ML^{-3}]=\frac{1}{[N^2]\,[T^2]} \qquad\qquad \ldots\ldots(R),$$

i.e., is of the dimensions of the square of an angular velocity divided by those of N^2. Naturally, Lord Kelvin having already taken N as dimensionless, the density came out with him as the square of an angular velocity. Inversely, if we take the density as proportional to the square of an angular velocity (which involves the hypothesis $[N]=[G]$) the force comes out as

$$[F]=\left[\frac{N^2}{G}\right]\frac{L^4}{T^4} \qquad\qquad \ldots\ldots(S).$$

If now we set $[N, G]=MT^2L^{-3}$, both density and force are of their normal dimensions. The inference is that neither of Lord Kelvin's conclusions was correct, and that this great physicist was, in fact, talking nonsense. Once the fact is grasped that in the application of mathematics to physical problems it is not justifiable to equate quantities the dimensions of which may be different even after units have been chosen so as to make the numerical value of a constant unity, then the difficulties disappear in the electromagnetic problem also. For here we must write for the force between poles, say, where β is a force-constant and μ the permeability,

$$F_1=\frac{mm'}{\mu\beta r^2} \qquad\qquad \ldots\ldots(T),$$

while for the electrical case we have (α another force constant, k the dielectric constant)

$$F_2=\frac{ee'}{k\alpha r^2} \qquad\qquad \ldots\ldots(U).$$

Proceeding on the assumption (the only possible assumption for those who accept any sort of electron theory) that both μ and k are merely *ratios*, and therefore dimensionless, we have, comparing the values of $[e]$ from equation (U) and from a dimensional equation expressing the first law of electrodynamics,

$$[\alpha^{-\frac{1}{2}}\beta^{-\frac{1}{2}}]=LT^{-1} \qquad\qquad \ldots\ldots(V),$$

and for this to agree with the second law of electrodynamics (B and H being taken as of the same dimensions) it follows that

$$\left.\begin{array}{l}[\beta]=\text{dimensionless}\\ {[\alpha]=T^2L^{-2}}\end{array}\right\} \qquad\qquad \ldots\ldots(W),$$

so that there is now no longer any question of $1/\sqrt{\mu k}$ being the velocity of electromagnetic waves, which is given instead by $1/\sqrt{\alpha\beta}$. Now whether in (U) we choose $\alpha=1$ (e.s.u.) or $\mathbf{c}^{-2}$ (e.m.u.) the dimensions of α are "still there", while k is without dimensions in both cases.

Since α has the dimensions T^2L^{-2}, we may conjecture that $\alpha = \mathbf{c}^{-2}$ in the electromagnetic system and $[\mathbf{c}^{-2}]$ in the electrostatic, where the actual amount of α will be unity. β, on the other hand, is unity in the e.m. system and $\mathbf{c}^{-2}[\mathbf{c}^2]$ in the e.s. system.

Electric displacement, having the dimensions of a charge per unit area, is no longer, as with Maxwell, of the same dimensions as the electric intensity E in one system of units, and of different dimensions in another, but now it is of different dimensions to E independently of the units*:

$$D = \alpha k E = \frac{kE}{[\mathbf{c}^2]} \qquad \ldots\ldots(\mathrm{X}),$$

in which, if the units are e.m.u., the brackets should be removed. The displacement, like the charge, is reduced in numerical value in the ratio 1 : $\mathbf{c}$ as we change from e.s.u. to e.m.u.; the electric intensity, on the other hand, is increased in the ratio $\mathbf{c}$: 1; but as far as "dimensions" go there is no longer any change. Hence Maxwell's difficulty † that (3)

"The electrostatic system of measurement . . . is independent of and incompatible with the electromagnetic system"

now seems to be adequately cleared up. In making a change-over from one system to the other all we are now doing is to alter the magnitudes of the units. The qualitative features of *all* quantities (including inductance, of dimensions L, and capacitance of dimensions T^2/L) remain unchanged by our choice of units, just as do μ and k, which remain always pure numbers.

REFERENCES

(1) W. H. J. Childs, *Physical Constants* (Methuen, 1934), p. 63.
(2) Kelvin, *Popular Lectures and Addresses*, vol. i, p. 104, 1891.
(3) *Scientific Papers* (Cambridge, 1890), p. 569.

Dr. P. Vigoureux. Students cannot fail to be deterred by the complication and asymmetry exhibited by the equations of electromagnetism as usually presented to them. For whereas electric current density is the product of electric intensity and conductivity, electric induction is obtained by dividing by 4π the product of electric intensity and permittivity. Moreover, although magnetic induction is written down simply as the product of magnetic intensity and permeability, things are in reality more complicated, for magnetic intensity is itself expressed as 4π times current per unit length, instead of merely current per unit length. Thus three equations, representing three analogous phenomena, are yet different in form.

As for Maxwell's equations as ordinarily written down, they are so complicated that it will, I think, be generally agreed that only those who continually use them can remember them by heart. Thus the student is presented with a medley of unpalatable mathematical expressions whose complication keeps hidden from him the great simplicity of the phenomena.

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* Thus, I maintain, Maxwell was mistaken if he ever supposed D and E to be quantities of the same kind in the same sense as I think B and H are.

† For this and other objections see Prof. A. O'Rahilly, *Electromagnetics* (Longmans, 1938), p. 700.

By employing only the units in common use, viz. the ampere, volt, ohm, coulomb, henry, farad, metre, kilogramme, second, joule, watt, which are realities to the student, and the corresponding unit of magnetic flux, viz. the volt-second or weber, we obtain, as shown in the table, a system of equations easily committed to memory, in which all the quantities are expressed in units familiar to the student, who cannot fail to be favourably impressed by its beautiful symmetry and simplicity. The nefarious two, three or more "systems" of units are not inflicted on him, nor does he become involved in speculations on the "nature" of B or H or any other quantities. The above presentation makes him realize that their "nature" depends on how they are defined, and that choice of definitions is dictated purely by convenience, not by immutable laws of nature.

(amperes/sq. metre)	$\mathbf{J}=\gamma\mathbf{E}$	(volts/metre)	Maxwell's equations
(coulombs/sq. metre)	$\mathbf{D}=\kappa\mathbf{E}$	(volts/metre)	curl $\mathbf{H}=\dot{\mathbf{D}}+\mathbf{J}$
(webers/sq. metre)	$\mathbf{B}=\mu\mathbf{H}$	(amperes/metre)	$-$curl $\mathbf{E}=\dot{\mathbf{B}}$

Velocity of propagation of electromagnetic waves	$=1/\sqrt{\kappa\mu}$	metres/second
Energy of electric field	$=\frac{1}{2}\kappa E^2$	joules/cu. metre
Energy of magnetic field	$=\frac{1}{2}\mu H^2$	joules/cu. metre
Poynting flux	$\mathbf{S}=[\mathbf{E}, \mathbf{H}]$	watts/sq. metre
Force between electric charges Q, Q' coulombs	$f=\dfrac{QQ'}{4\pi\kappa r^2}$	newtons*
Force between magnetic poles Φ, Φ' webers	$f=\dfrac{\Phi\Phi'}{4\pi\mu r^2}$	newtons

* The newton is the force which gives to a mass of 1 kg. an acceleration of 1 metre per sec. per sec.

The practical units are such that for air κ and μ have the numerical values $8{\cdot}86\times10^{-12}$ and $1{\cdot}256\times10^{-6}$ respectively, or, putting the units in evidence, 8·86 micromicrofarads per metre and 1·256 microhenry per metre; for other substances the values are equal to the above multiplied by the specific inductive capacity and specific permeability respectively.

Exception is sometimes taken to the last two equations in the table: it is said that removal of 4π from certain equations has resulted in its finding its way into others. But this is not an accurate version of the facts: 4π has been removed from the places where it ought never to have appeared, to wit the first three equations, which represent problems whose geometry is cartesian, and it has been restored to where it ought to be, i.e. in the last two equations, which deal with point charges, therefore with a spherical problem, in which 4π very naturally appears as the area of the unit sphere.

In a presentation of electromagnetic theory it is possible to take as starting point general notions like those of energy in the field and Poynting flux and deduce the particular cases; or one may start from a number of simple experimental facts and derive the general laws. In the latter method, if among the experimental facts the reaction between electric charges and between magnetic needles are included, the 4π in the equations need cause no uneasiness. The student realizes that the geometry of the problem is spherical, and anyway the first three equations are soon derived, and he sees in their perfect symmetry and simplicity another powerful reason, one of convenience this time, for the previous introduction of the 4π. Nor is there any reason why one should straightway assign simple arbitrary values like 1 or 4π to the constants of proportionality of the two basic equations. In the first instance let the constants be expressed in terms of the units employed, just as the constant of gravitation is expressed in terms of the units of length, mass and time in common use. The fixation and preservation of the units is a technical question, which should be dealt with at a later stage.

I venture to claim that universal adoption of the method here outlined, the method of G. Giorgi and G. A. Campbell, for the presentation of the equations of electromagnetism, would be of great benefit not only to students but to mathematical physicists and research workers as well as engineers.

Mr. J. Nicol. I should like to comment on three points raised by Dr. Burniston Brown's most stimulating paper.

(1) I cannot agree with his criticism of Lorentz for mixing macroscopic and microscopic methods in considering the Mosotti problem. Macroscopic methods will not give a solution, but there is no reason why they should not be used as a short cut to carry us as far as possible, leaving only the investigation of the effect of a small sphere of material to be done by microscopic methods. Macroscopic methods by themselves predict the existence of a self-demagnetizing force inside a magnet, but the Lorentz analysis would add a magnetizing term $4\pi I/3$, which may be compared with Ewing's theory of the stability of a polarized array of magnetic molecules or with Weiss's assumption of the existence in each of his domains of an internal molecular field proportional to and in the same direction as the magnetization.

(2) I consider that no "New treatment of Electric and Magnetic Induction" can be useful to teachers if it is restricted to materials for which $I/H=\kappa$ is a constant, since all their students except those in the special honours stage are concerned with the magnetization of iron and not with para- and diamagnetic phenomena. As a consequence of this treatment there is much confusion in the textbooks as to the energy of magnetization. The student is left in doubt whether $H.\delta I$ or $I.\delta H$ is the right expression when κ is not constant. I think it is correct to teach that $-I.\delta H$ is the work done per unit volume in transporting a small piece of material into a stronger field, but that this $=-\delta(H.I)+H.\delta I$, where the first term represents the increase in the potential energy of the distribution of magnetic poles in the field and the second term the actual work done in magnetizing the material. [Reference may be made to a paper by E. C. Stoner (*Phil. Mag.* May 1937).]

(3) The proof from the conservation of energy that $E = -dN/dt$, quoted by Dr. Brown from Starling's *Electricity and Magnetism*, is unsatisfactory because it neglects the fact that the configuration of the system is altering as the slider moves and that potential energy may be accumulating. If Pidduck (*Treatise on Electricity*, 1925, p. 161) were right in his statement that a current circuit in a magnetic field possesses potential energy $= i \times N$, the proof would fail. It can, however, be made satisfactory by arranging that the rails form a pair of concentric circles and the slider part of a radial arm pivoted at their common centre. The arm will rotate if a uniform magnetic field exists perpendicular to the plane of the circles, and after any whole number of rotations there is no alteration in the configuration, and so the energy supplied by the battery may justifiably be equated to the work done by the rotating arm + the heat generated in the circuit (J. J. Thomson, *Elements of Electricity and Magnetism*, 1897, p. 456).

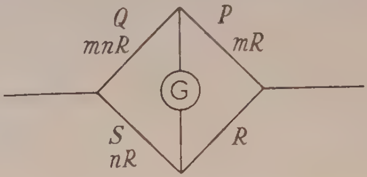
Dr. D. Owen. It is very desirable to bring out the analogies between processes in the magnetostatic and electrostatic fields. It seems unfortunate, from the teaching point of view, that the same use is not made of polarization, or electric moment per unit volume, in the electrical case, as of intensity of magnetization in the magnetic case. Instead of this, following Maxwell, it is usual to introduce a special quantity, electric displacement, D, which actually differs from "electric induction" (to be taken as the analogue of magnetic induction, B) only by the numerical factor 4π. It is important too that authors of textbooks should restrict the term *electric polarization* to denote electric moment per unit volume and not make it synonymous with electric displacement.

The following comments on teaching methods are made on points outside the scope of Dr. Brown's paper:—

(1) The practice should be abolished of supposing that the position of a pole of a bar magnet may be found as the intersection of two lines of force drawn near the pole. It has no warrant in theory, and gives quite wrong results in practice.

(2) The teaching of elementary electrostatics can be rendered more definite if charge, potential and lines of force are all three associated at the same time, instead of attempting to interpret phenomena in terms of two only of these. For example, in explaining the phenomena of electrostatic induction, suppose the conductor on which is a positive inducing charge to be at 30,000 volts, and that an insulated uncharged conductor is brought towards it. Lines of force leaving the former will end on the latter, while lines of force leaving the latter will end on the walls or boundary of the room. The second conductor will be at an intermediate potential of, say, 20,000 volts. On connecting it momentarily to earth its potential drops to zero, and at the same time the positive induced charge disappears. But this conductor differs now in potential by about 30,000 volts from the first conductor, hence the negative induced charge will be greater than at first. This fact may of course be demonstrated by a proof plane and electroscope.

(3) The theory of the balanced (or nearly balanced) Wheatstone bridge is made far simpler if, following Schuster and Callendar, we denote the resistances of the four arms by R, nR, mR and mnR, R being the resistance to be measured ;



and if, in order to find the out-of-balance current through the galvanometer when R is changed by a small amount δR we suppose an e.m.f. $i\delta R$ to act in the R arm (i denoting the current through R), the battery circuit being considered broken. Thus the galvanometer current can be quickly deduced, and it is readily shown, for example, that the resistance to which the galvanometer should be wound for maximum sensitivity is given by the expression $R\,.\,\frac{n(1+m)}{1+n}$, or, denoting the resistances of the arms (in the order already mentioned) by R, S, P and Q, by the expression $R\,.\,\frac{Q+S}{R+S}$.

Mr. A. G. Warren. The dispute over dimensions is artificial. Though it is more acute in connection with electricity and magnetism than in other fields, the possibility of confusion is equally present in mechanics. It is a common failing to read into an equation much more than is justifiable, until a stage is reached when it is accepted that force is equal to mass multiplied by acceleration in the same sense that momentum is equal to mass multiplied by velocity. Clear thinking is the only remedy for avoiding such absurdities.

When it is recognized that equations can only express relations between measures of certain selected properties of things, it is seen to be natural that the dimensions of a quantity should not be unique. The dimensions of mass in terms of force, length and time naturally differ according to whether one views the mass in its inertial or gravitational aspect; similarly the dimensions of an electric charge must differ according to whether one is measuring the mass by its capacity to repel similar charges or by its capacity to produce a magnetic field when it is in motion. Any attempt, by the introduction of coefficients having dimensions, to make the two measures commensurable is philosophically unsound.

The dimensions of a quantity may be decided by choice of a definition. A case in point is that of B. Personal preference may decide its dimensions There is much to be said for its dimensions being the same as those of H, and (with Pidduck) considering H as that part of B derived from a potential.

It is common to deplore the unfortunate choice of units which has resulted in similar equations in electrostatics and magnetism being dissimilarly expressed. Admittedly it is annoying when making a hurried calculation to be uncertain whether the answer is correct, 4π times too big, or 4π times too small. But it is perhaps more annoying to be led to a conclusion entirely wrong in kind by an over-emphasis on the analogy between electrostatic and magnetic phenomena when the analogy is deficient in important respects.

The magnetic force is always a solenoidal vector. The electric force is not. As a simple illustration, the electric flux of a charge is q, the nett magnetic flux of a pole is not m, or even $4\pi m$, but zero. We have no experience of magnetic poles from which flux emerges without also arriving, and although we may, after investigation, guess at the behaviour of such things, if they existed, we have no right to assume them as fundamentals. If one assumes the possibility of a pole having an emergent flux $4\pi m$, it is easy to show that its reaction with an electric circuit is entirely different from the reaction between two electric circuits, and, therefore, such a pole could not be electric in origin.

The magnetic pole of the textbooks is one of those convenient inventions which may be used on the unsuspecting student to derive certain results already known to be true. Its use in unfamiliar fields may lead to entirely false conclusions.

AUTHOR'S reply: The classical theory has the great merit of being independent of fluctuating atomic or molecular hypotheses. For this reason I prefer not to assume, as Mr. Benham suggests, that polarization is due to the rotation of permanent doublets. The method I have indicated does not assume that the doublets are molecules, nor does it make any assumption about the mechanism of polarization.

I have attempted in this paper to demonstrate that the μ in

$$(a)\ F=\frac{m_1 m_2}{\mu d^2},\quad (b)\ \mu=B/H,\quad (c)\ \mathrm{E}=\partial/\partial t\left(\iint \mu H_n\, ds\right)$$

is the same. I am not quite sure that this would be equally easy if Mr. Cosen's ingenious derivation of B and H were employed. Of course, I expect that electrical engineers will say that their students are not concerned with (a) and (b), and Pidduck's text-book certainly does not even mention μ. But the treatment of Maxwell's equations is then bound to be unsymmetrical and incomplete.

In reply to Mr. Nicol's third point: I think the proof is satisfactory; the configuration of the system *is* altering and the potential energy *is* accumulating, and this does change from $i \times N$ to $i \times (N+\delta N)$. This is the result of the work done in enlarging the circuit and comes from the battery. I think, however, that the proof might be clearer if it were emphasized that the e.m.f. e, which maintains the current i while the slider is moving, is higher than that which would be necessary to maintain i with the slider stationary.

The reason for retaining cavities is an epistemological one; they give meaning to the concept of *force in a medium* (cf. my article in *Nature*, **145**, p. 789, 18 May 1940), and all attempts to develop classical physics without the concept of force have failed, as, I think, they must inevitably do.

A PRECISION FREQUENCY-METER OF RANGE 0 TO 2000 Mc./sec.

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ABSTRACT. The paper describes a new method and equipment for the measurement of frequencies in the range 0 to 2000 Mc./sec. Frequencies less than 30 Mc./sec. are compared directly with a signal obtained from oscillators controlled by the standard, and the accuracy of the measurement is limited only by that of the standard, which is ± 2 parts in 10^8 if the primary quartz clock at the National Physical Laboratory is used. Frequencies above 30 Mc./sec. are compared with harmonics of an uncontrolled oscillator of which the frequency is less than 30 Mc./sec. and is measured by the direct method. The accuracy is then governed by the stability of this oscillator during the measurement and is of the order of 1 part in 10^6.

The method involves a simple extension of the principles of frequency multiplication and division. From the standard frequency f, controlled oscillations are produced at frequencies $\frac{a}{b} \cdot f$, where a has the whole-number values between 8 and 11 and b has values between 9 and 22. A scale of frequencies $n \frac{a}{b} \cdot f$ is thus obtained, where n is the order of harmonic, and the various possible combinations of n, a and b give frequencies on this scale at an average spacing of 2·4 kc./sec.

The apparatus is very simple in construction and operation. It does not, however, permit of direct frequency reading on a decade scale, but gives the values of n, a and b, the actual value of frequency then being read from tables.

The paper includes notes on the accuracy of harmonic methods of frequency measurement and on the division of high frequencies.

§ 1. INTRODUCTION

Modern methods of measuring radio frequencies are based on the principles governing the control of oscillators at the frequency of a standard oscillator and the production of harmonics, which were developed by many experimenters between the years 1905 and 1920, and were first applied to modern apparatus by Pierce(1) and Dye(2). The standard in terms of which all the frequencies are measured is a mechanical vibrator such as a tuning fork or quartz crystal, which is maintained in continuous vibration in a valve circuit. The frequency f of the standard is measured by direct reference to the mean solar second by well known methods(3) which need not be described here. Valve oscillators designed to be very rich in harmonics are controlled by the standard at the frequency f, and also, by introducing stages of frequency multiplication and division, at frequencies $\frac{a}{b}f$, where a and b are whole numbers. Series of frequencies $n\,af/b$, where n is the order of harmonic, are provided by the harmonics of the controlled oscillators, and unknown frequencies are measured on this scale of standard frequencies.

The measurement can be effected in two ways which may be called the interpolation and the direct methods. In the former a local oscillator, provided with a finely divided scale, is tuned to synchronism first with the signal of unknown frequency and then with the nearest standard harmonics on either side of it. The signal frequency is calculated by interpolation on the oscillator scale. Since the law of this scale is seldom linear or known with great accuracy, it is essential that the two standard frequencies should not differ by more than, say, 0·001 of the frequency being measured; and the accuracy is also limited by the instability of the local oscillator during the measurement. In the direct method a beat-note of audible frequency is produced by interaction of the signal and one of the standard harmonics, and the beat-note frequency is measured by comparison with an audio-frequency oscillator. If this is itself controlled by the standard, as in the design described elsewhere⁽⁴⁾ by the author, for instance, a direct comparison is obtained between the signal to be measured and a signal obtained from a combination of frequencies all controlled by the standard. The accuracy of the frequency measurement is then limited only by that of the standard. If the frequency of the beat-note is a very small fraction of the frequency being measured (as is the case for frequencies above, say, 10 Mc./sec.) or if the accuracy required is not high, a simple form of audio-frequency oscillator may be used. It is clear that for this method the spacing between consecutive standard harmonics must be not greater than about 10 kc./sec.

The practical difficulty of frequency measurement rests on the fact that a scale of standard frequencies must be produced with a small interval between neighbouring points and that at the same time it must be possible to separate and distinguish the individual points. Even with the most stable and selective tuned circuits it is difficult to select with certainty a harmonic of order higher than 100 for lower- and 50 for higher-frequency ranges. Different methods of overcoming this difficulty have been used in the various equipments which have been described. Some of these use high-order harmonics and stable tuned circuits or calibrated interpolation oscillators to determine the order of the harmonic. Others use a lower-order harmonic of an oscillator of a higher fundamental frequency. The gap between successive harmonics is then large, and standard points in this gap are sometimes provided by modulating the oscillator by a lower standard frequency. In Dye's original method, for instance, a 20 kc./sec. oscillator is modulated at 1 kc./sec., enabling the 20 kc./sec. gaps to be subdivided into 20 gaps of 1 kc./sec. None of the equipments so far described makes full use of the ease with which frequencies can be divided, only a few fixed values of b, usually

10 and 100, being used. The apparatus tends to be elaborate and demands careful construction and use.

The method described in this paper is a simple application of the principles outlined above, and differs mainly in that both a and b can be varied over a limited range. This results however in a considerable simplification in the design and construction of the apparatus and in the process of measurement. At the same time it is believed to represent an improvement both in frequency range and in accuracy attainable.

§ 2. DESCRIPTION OF THE METHOD

The method is best explained with the help of the block diagram of figure 1. The 100 kc./sec. multivibrator A is controlled by a quartz oscillator. The circuit D selects the 8th, 9th, 10th or 11th harmonic of A, and controls the multivibrator E at the same frequency (af where $a=8$, 9, 10 or 11). This is followed by another selector circuit F tuned to the same frequency as D and E. The multivibrator G is provided with a variable condenser by means of which its frequency can be varied between 50 kc./sec. and 100 kc./sec. It is coupled to the selector circuit F and, as the condenser is varied, is controlled in turn at each of the fixed frequencies af/b, where b has any whole-number value giving a frequency between 50 kc./sec. and 100 kc./sec. If for instance a is 10, then b may have any value between 20 and 10. By using the various possible combinations of a and b, G can be controlled at 35 different frequencies, and its harmonics, received in the receiver H, give standard frequencies throughout the range between 0·5 Mc./sec. and 30 Mc./sec., which are spaced on the average 2·4 kc./sec. apart. The spacing is of course not uniform and there are in this range two gaps of as much as 14 kc./sec. In spite of the close average spacing of possible harmonics, when G is set at any one controlled frequency there is a spacing of at least 50 kc./sec. between neighbouring harmonics and there is no difficulty in distinguishing between them. At frequencies up to 5 Mc./sec. the calibration of the receiver scale is sufficiently accurate to give the order of harmonic used. For higher frequencies, the circuits D, E and F are first set at 1 Mc./sec. and G at 100 kc./sec. The harmonics

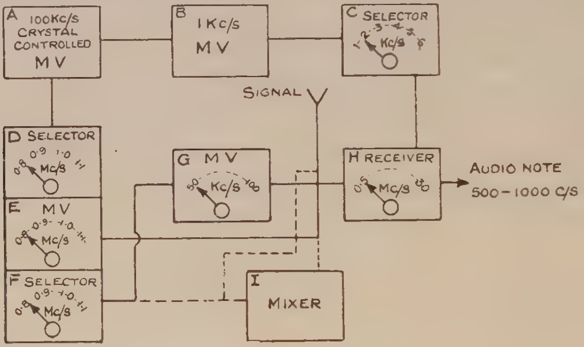

Figure 1. Representation of the equipment for measuring frequencies in the range 0 to 30 Mc./sec.

of F and G then determine each megacycle point on the receiver, together with the intermediate 100-kc./sec. points. A few seconds only are required to obtain on this scale a measurement of the signal accurate to 20 kc./sec., and this enables the order of harmonic to be determined. The beat-note produced between the signal and the nearest standard harmonic has on the average a frequency of 1·2 kc./sec., and in the two cases mentioned is as high as 7 kc./sec. It is difficult to measure high audio-frequencies with precision, and where necessary they are therefore reduced to a value less than 1 kc./sec. by being mixed with a note of 1, 2 . . . 6 kc./sec. obtained from the multivibrator B and tuned circuit C.

The process of measurement is very simple. The receiver, which is a commercial communication model, is tuned to the signal, D, E and F are set at 1 Mc./sec. (one control makes the three adjustments), and the frequency of G is varied throughout its range until a steady note is heard in the receiver. If no such note is heard it is because the signal frequency falls in a gap in the series of harmonics $n \cdot \frac{10f}{b}$ and D, E, F must be set at another value of a and the process repeated. It is seldom that more than two values of a need be tried before a suitably low beat-note is heard. The value of b is read on a scale which has four ranges corresponding to the four values of a. The frequency of the standard harmonic can be calculated from the values of a, b and n or may be more easily obtained from tables. The value of the beat-note is then measured, the frequencies from C being used if necessary. There are always at least two combinations of a, b and n which give an audible beat-note, and it is therefore easy to determine whether the frequency of the beat-note is to be added to or subtracted from that of the standard harmonic to obtain the signal frequency.

If the signal frequency is below 0·55 Mc./sec. (the lower limit of the receiver used), it can be mixed at I with an output from F, which changes the signal by a known amount (0·8, 0·9, 1·0 or 1·1 Mc./sec.) and enables it to be received and measured in the same receiver H.

For the measurement of frequencies higher than 30 Mc./sec., the harmonics of a local oscillator are used. The oscillator, of range 12 to 24 Mc./sec. and having a fine tuning control, is set so that one of its harmonics is in synchronism with the signal frequency. The frequency of the oscillator is then measured by the method described above. The order of the harmonic is found by measuring the oscillator frequency corresponding to two different orders of harmonic. If F is the signal frequency, f_n the frequency of the oscillator corresponding to the nth harmonic, and $f_{(n+x)}$ that corresponding to the $(n+x)$th harmonic, then

$$nf_n = (n+x)f_{(n+x)} = F,$$

$$n = \frac{xf_{(n+x)}}{f_n - f_{(n+x)}},$$

$$F = \frac{xf_n f_{(n+x)}}{f_n - f_{(n+x)}}.$$

The approximate values of frequency as read on the calibrated scale of the oscillator serve to determine n and, when an accuracy of 1 in 10^3 is sufficient, the frequency of the signal. The fine tuning control enables measurements to be made by the interpolation method with an accuracy of 2 parts in 10^5. If the direct method is used, the accuracy is governed by the stability of the oscillator during the measurement and is of the order of 1 part in 10^6. Frequencies up to 2000 Mc./sec. may be measured by this method.

§ 3. PRACTICAL DETAILS

The quartz vibrator controlling the multivibrator A may either be housed separately and maintained in operation under the stable conditions necessary for the highest frequency stability, or it can form part of the multivibrator circuit, in which case the whole equipment is portable. The vibrator used in the portable form of the equipment is a bar of the type described by Scheibe and Adelsberger (5), but is in a simpler form of mounting and is capable of a stability of 1 part in 10^7 over a period of several hours. The circuit diagram, figure 2, shows the quartz vibrator as part of the multivibrator circuit. If the quartz is housed separately, the multivibrator is of the normal form and is controlled by a voltage applied to the grid of the first valve. At the National Physical Laboratory the quartz clock (6) is used as the standard and the accuracy of measurement is then ± 2 parts in 10^8.

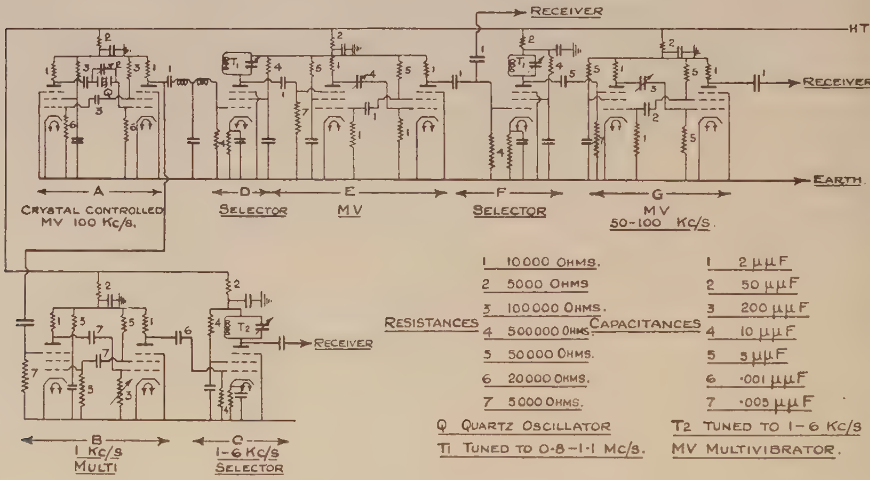

Figure 2. Circuit of equipment for range 0 to 30 Mc./sec.

The four multivibrators used in the equipment are of the same type, using H.F. pentode valves, and are controlled by a small voltage applied to the suppressor grid of one of the valves. A number of different forms of multivibrator using both triodes and pentodes were tried and all were found to operate fairly satisfactorily, but the form adopted gave the best performance as regards ease of synchronisation and strength and uniformity of harmonics. The full circuit diagram, figure 2, with details of the more important components may therefore be of use, though it should be remembered that the values of the components depend to some extent on the particular valves employed.

The quartz vibrator is coupled loosely to the multi-vibrator circuit, by a capacitance of $1\mu\mu$ F., but the control is still very strong. The small variable condenser across the quartz electrodes enables the frequency to be varied by a few parts in 10^5, so that it can be set from time to time to that of a reference standard. The components of the 50 to 100 kc./sec. multivibrator G and the controlling voltage applied to it need to be selected rather carefully in order that its frequency may be controlled uniformly at the values af/b. Between each pair of controlled frequencies there is a very small uncontrolled range. Varying beat-notes are sometimes produced in the receiver from harmonics of these uncontrolled regions, but there is no danger of confusing them with the steady notes obtained from the controlled oscillations. It is important to ensure by screening and filter circuits that no harmonics of the 100 kc./sec. multivibrator A reach the receiver. It was found that 1 Mc./sec. was near the limit of multivibrator oscillations when ordinary H.F. pentodes were used and that the harmonics were consequently weak. Valves with small inter-electrode capacitances, such as acorn pentodes or EF 50s (Mullard), were therefore used for the multivibrator E.

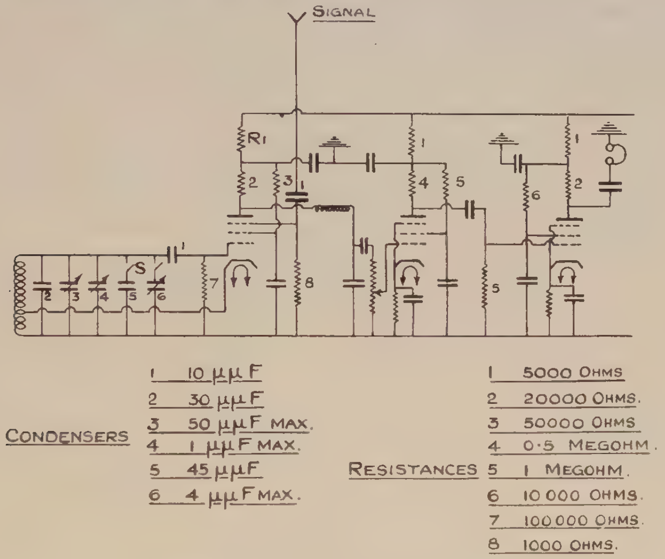

Figure 3. Circuit of oscillator and radio-frequency amplifier used for extending frequency measurements to 2000 Mc./sec.

It might be thought that the series of harmonics obtained from 35 different frequencies af/b would be difficult to tabulate; but since the series can be represented as

$$af\left[\frac{n'}{b}+\left(1+\frac{n'}{b}\right)+\left(2+\frac{n'}{b}\right)+\ldots\ldots\right],$$

where n' has the values 1 to b, it is clear that the same subdivisions of the interval recur between each pair of values of naf. All the possible frequencies can therefore be contained in tables occupying only a few pages, and very little time is required to find the frequency corresponding to given values of a, b and n, or to find the values of a, b, giving the harmonic nearest to a given frequency.

The circuit used for extending the frequency range to 2000 Mc./sec. is shown in figure 3. The oscillating circuit contains two variable condensers, one of which is for fine tuning and covers a frequency range of about 200 kc./sec. To secure greater openness of scale, the band 12 to 24 Mc./sec. is covered in two ranges. The range switch S adds a fixed capacitance and also connects an additional variable capacitance to the fine tuning condenser so that the band covered by this remains of the same order. A linear frequency scale is used to facilitate interpolation. The signal is applied to the suppressor grid of the oscillator valve which acts as oscillator, mixer and detector. The audio note produced is amplified by a two-stage audio-amplifier.

The equipment is mains-driven throughout.

§4. NOTE ON THE DIVISION OF HIGH FREQUENCIES

Equipments for measuring frequencies up to 1 Mc./sec., such as that designed by Dye, are in fairly common use and are free from some of the practical difficulties experienced at higher frequencies. Some experiments have been made by the author with the object of sub-dividing frequencies in the range 1 to 30 Mc./sec. so as to produce values within the range 0·5 to 1 Mc./sec., thus enabling existing equipment to be used for their measurement. Though this method was not ultimately adopted, it was under certain conditions quite practicable, and the results obtained may be of use in connection with frequency measurement.

The signal was applied through a capacitance of 2 $\mu\mu$F. to one anode of a multivibrator similar to E, whose frequency could be varied between 0·5 and 1 Mc./sec. A receiver was tuned to the signal and the frequency of the multivibrator was varied until a harmonic from it was heard to synchronise with the signal. Control of the multivibrator was recognised from the nature of the sounds in the receiver. The frequency of the multivibrator was then measured, and the factor of division could be ascertained from the calibration of the receiver. The voltage necessary for controlling the multivibrator was 0·5 volt at frequencies of the order of 30 Mc./sec. and 0·1 volt at frequencies less than 10 Mc./sec. A low-power local oscillator loosely coupled to the multivibrator produced satisfactory control, but it is clear that a distant signal would need considerable amplification.

§5. NOTE ON THE ACCURACY OF HARMONIC METHODS OF FREQUENCY MEASUREMENT

In practically all methods of radio-frequency measurement, the signal is measured on a scale of standard frequencies which are obtained from the standard oscillator by various stages of multiplication and division involving the use of a number of controlled oscillators. Although it would usually be noticed if one of these fell out of control during a measurement, changes in the phase relationships between the controlling voltage and the controlled oscillator might occur while the oscillator is controlled. Such phase changes would be interpreted as variations of frequency during the time occupied by the change. Those who have used frequency-measuring equipment know from the consistency of the results that any such effect must be small, but it was thought desirable to make quantitative measurements to determine what precautions, if any, should be taken to guard against errors due to such causes. Two standard quartz oscillators were used and their frequencies, of nominal values 20 kc./sec. and 100 kc./sec., were compared by two different methods. In method 1 outputs from the two oscillators were applied directly to the plates of a cathode-ray oscillograph and the frequency difference was measured by timing the movement of the Lissajous figure produced. In method 2 the frequencies were compared after the sequence of tuned circuits and controlled oscillators shown in figure 4. The beats produced in the receiver were measured during the same interval of time as that occupied by the measurement of method 1. This was about 15 seconds, so that the measurements represented average values during such periods.

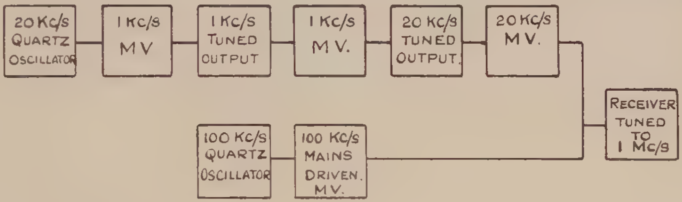

Figure 4. Representation of equipment for checking the accuracy of harmonic methods of frequency measurement.

Under steady conditions the two measurements agreed on the average to ±2 parts in 10^9 and the differences between individual measurements were within ±10 parts in 10^9. The accuracy of the individual measurements was about ±5 parts in 10^9. The various controlled oscillators were then switched off and allowed to cool. When they were switched on again, measurements were begun as soon as they fell into synchronism. Under these conditions maximum differences of 2 parts in 10^6 were obtained for about 15 seconds, but after 3 minutes the measurements were always in agreement to 10 parts in 10^9. The condenser of the 1-kc./sec. multivibrator was varied uniformly at the rate of 5 $\mu\mu$F. per second; a steady frequency-difference of 1 part in 10^6 was produced.

The results show that under ordinary conditions no errors greater than 10 parts in 10^9 should be produced by random circuit fluctuations so long as the circuits have been switched on a few minutes before any measurements are made. No particular precautions were taken to stabilise the batteries and mains supplies used for driving the equipment.

ACKNOWLEDGEMENT

The author wishes to thank Mr. W. Wilson for his helpful suggestions on questions of circuit design, and in particular for suggesting the multivibrator circuit finally used.

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CLAY-WATER RELATIONSHIPS

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ABSTRACT. It is shown that if the plastic mixture of clay and water be subjected to a steady pressure between porous pistons, the applied pressure P is related to the equilibrium moisture-content M by the expression $P=\alpha e^{-\beta M}$, and the aqueous conductivity C obeys a similar relation $C=\epsilon e^{\phi M}$. The validity of the assumption that the water movement during drying is associated with the pressure P, and that of the extrapolation of the empirical formulae for P and C to moisture-contents below those possible experimentally, is demonstrated by a comparison of experimental and calculated moisture-content distributions in clay bars drying from one end only.

The variation in C cannot be explained if the viscosity of the water in the mixture is the same as that in bulk, but the experimental data on a wide range of clays are in agreement with the supposition of an anomalous viscosity, very large at the solid-liquid interface and decreasing exponentially with distance from that interface. This is attributed to an intensity of crystal structure in the water which decreases from the interface in a similar manner.

§ 1. INTRODUCTION

Clay, in general, consists of hydrated aluminium silicates. The term, in the industrial sense, covers a very wide range of minerals from hard shales to bentonites, which, though comparable chemically, exhibit surprising variation in their properties. The predominant component of most of these is kaolinite or silicates of similar structure(1). Bentonite clay consists largely of montmorillonite, which has received much attention through its property of increasing and decreasing its interlamellar distance with change in moisture-content(2). This mineral, however, is generally present only in very small quantities.

The crystal structure of the silicates indicates that they will show cleavage, though not in any pronounced degree. In commercial clays the particle size may range from $\frac{1}{4}$ in. or more in diameter down to about 500 A.(3) Particle size is one of the most important factors influencing the behaviour of a clay, and it is to be regretted that there is no method known at present for its direct determination. All methods depending on Stokes's law are open to suspicion, since the clay suspension contains groups of flocculated particles and the particle size measured is not that appertaining to the plastic state.

The clay industry is to a large extent dependent on the phenomenon of plasticity which is exhibited by the mixture of the ground mineral with a suitable proportion of water. Other important properties, both from the fundamental and the manufacturing points of view, are drying shrinkage and general drying behaviour.

If an excessive amount of water be added to the ground mineral, a fluid slip results. In this condition it is used for casting articles, particularly pottery, in plaster moulds, and the fluidity of these slips, some of which show thixotropy, has therefore received attention, though mainly from the manufacturing angle. Reduction of moisture-content produces first a soft and sticky solid which hardens and loses its stickiness as the moisture-content is progressively reduced. The condition in which the moisture-content is just below that at which the material sticks to the fingers is known as that of maximum plasticity or best working consistency, and, as the name implies, it is in this state that the clay is best moulded and articles made by hand. This is, roughly, the maximum moisture-content with which this paper is concerned.

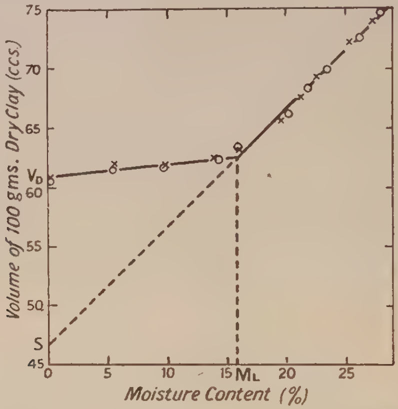

Figure 1. Volumetric shrinkage of Clay 28, showing meaning of S, M_L, and V_D.

As the clay dries from this condition, there is first a shrinkage period where the volume change is equal to the volume of water lost. This shrinkage ceases at a comparatively high moisture-content, and during the removal of the remaining water there is only a very small change in volume, usually a decrease, although an expansion has been found in a few cases. There is in general a short transition period between the two stages, but the extent of this is influenced by experimental technique, and it may largely be attributed to unavoidable differences in moisture-content between the interior and exterior of the test piece. Norton(4), using thin-walled cylinders, found a sharp cessation of shrinkage in some cases.

Moisture-content is taken as a percentage on the dry weight of clay, and when the volume is calculated for the mixture containing 100 gm. dry weight of clay, the slope of the shrinkage curve is unity. The shrinkage behaviour of a clay may then be denoted by three terms, S, M_L, and V_D, the significances of which are apparent in figure 1.

These shrinkage curves are readily obtained by a simple mercury-displacement method. The moisture-content at which shrinkage ceases is associated with the change in colour, and this condition is known as *leather hard*. This is the lowest moisture-content with which we are concerned, since a definite change of structure takes place at this point:

The author has shown elsewhere (5) that the rate of drying of a moulded article under constant conditions of temperature, humidity, and air velocity may be divided into two periods corresponding to the shrinkage. During the first of these the rate is constant per unit surface area, i.e., independent of the moisture-content. It is of the same order, and obeys the same laws, as that of a free water-surface. When shrinkage ceases, water can no longer flow freely to the surface of the article and the air-water interface retreats into the clay, evaporation taking place by diffusion through the pores. The rate of drying falls off continuously as the lengths of the paths of diffusion increase, and as the vapour pressure is decreased by high curvature, until equilibrium with the atmosphere is reached at a comparatively low moisture-content.

During the drying of any article of size, cracking will occur if the rate of removal of the water be too great. This is due to differences in shrinkage associated with differences in moisture-content between the surface and the interior. Since this cracking is the source of considerable loss in the industry, the materials used in this investigation have been restricted to commercial clays of which sufficient quantities were available for parallel experiments on the drying of large blocks.

The shrinkage behaviour has been readily explained by Mellor (6), Norton and others by the supposition of water films between the surfaces of the clay particles. During drying, the shrinkage of these films results in the volume change being equal to that of the water lost until the particles come into contact, and the shrinkage ceases. The maximum thickness of these films under a pressure of 8 kg.-cm.$^{-2}$ has been estimated by Norton and Hodgdon (7) as from $2{\cdot}1 \times 10^3$ A. in a china clay, and $2{\cdot}4 \times 10^3$ A. in a ball clay, to $3{\cdot}1 \times 10^3$ A. in a brick clay, and $3{\cdot}3 \times 10^3$ A. in bentonite. Their figure is therefore about 3000 A. for ordinary clays in what may be considered to be about the state of maximum plasticity. It may be noted that the attractive force between two parallel plates due to surface tension has a value of 8 kg.-cm.$^{-2}$ when the distance of separation is 2000 A. Opinions vary, however, as to the actual value of the thickness of these films. From figures given by Mattson (8), an estimate of 700 A. can be obtained for a sodium-saturated soil. It may thus be said that the thickness of these films is of the order of 1000 A.

Houwink (9) shows by calculation that the maximum thickness of the solvation films on the clay surface will be of the order of 25 A., a figure in fair agreement with that of 40 A. obtained by Mattson. This is considerably lower than the estimated thickness of the water films, and the cause of these large distances must be looked for elsewhere than in solvation alone. Mattson advances the theory that the particles are surrounded by a film of osmotically imbibed water in addition to the adsorbed layer. This theory is, however, of a type to which objections have been raised (10).

The plastic substance may preferably be considered as a continuous water phase in which the clay particles, by mutual repulsion due ultimately to their surface charge, constitute a network of forces which gives the material its rigidity.

Each particle is then in a position of minimum potential energy in a way somewhat analogous to the atom in the crystal structure. The application of shear to such a system will cause movement of the particles over one another, but on the removal of the force a new and equally stable system will exist, enabling the material to retain its deformed shape. This is, in general terms, the phenomenon of plasticity.

That the repulsion between the particles exists between their surfaces rather than their mass centres is to be expected from the fact that their diameter is, at the least, of the same order as, and, in general, much greater than, the distance between their surfaces. Confirmatory evidence is found in the sharp cessation of shrinkage in a thin-walled cylinder, indicating that with uniform moisture-content the distance of separation of the surfaces, irrespective of particle size, tends to be uniform. This equality of distance between the particle surfaces, together with the plate-like shape of the grain, will account for several well-known properties. Linear shrinkage is affected by previous distortion of the clay and is different in the three directions in the column of clay extruded from a pug(11). A bar of clay may be bent and straightened, but on drying and firing will be found to have a curvature in the same direction as that originally given. Handles of articles thrown on the potter's wheel must be attached, not vertically, but at a slight angle, so that after firing they will be straight. These phenomena are due to a regularity of orientation of the clay particles produced by shear.

In natural clays, fortuitous packing of particles of very different sizes does not produce the ideal state of perfect uniformity of the distance of separation of the particles. Areas will exist where the conditions of minimum potential energy are not realised, somewhat analogous to the " dislocations " in the crystal structure. The disappearance of these areas due to rearrangement of particles is considered to explain the shear hardening which clay exhibits on deformation.

The reduction in moisture-content, and consequently of the distance of separation of the surfaces, is accompanied by an increase in the repulsive pressure. This intensification of the force system is immediately reflected in the hardening of the material.

During the drying of a moulded article, the repulsive force between the particle surfaces provides the pressure difference necessary to cause the flow of water from the interior to the surface where it is evaporated. Thus, for linear flow in a bar drying from one end only, if δP be the difference in the pressure of repulsion associated with a difference in moisture-content δM over a distance δx, the pressure gradient is $\partial P/\partial x$ and the solution of the equation

$$\frac{\partial}{\partial x}\left[C\frac{\partial P}{\partial x}\right]=\frac{\partial}{\partial t}\left(\frac{S}{S+M}\right) \qquad \ldots\ldots(1)$$

will give the moisture-content distribution throughout the bar at any time t, where $C=f(M)$ is the aqueous conductivity * of the clay, defined as the rate of flow through unit cube in unit time under unit pressure difference.

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* The term permeability, although commonly used in this sense, has not been chosen as it already has a well-defined meaning in the clay industry.

If a plastic clay be subjected to a steady pressure between porous pistons, equilibrium will be reached when the repulsive pressure between the particles is equal to that applied. The variation of the pressure P with moisture-content may thus be measured, and the moisture-content controlled during the determination of the aqueous conductivity. The rapid change of the pressure P and its very high value at the lower moisture-contents limit the range of experiment, and render the use of indirect methods necessary. Comparison of solutions of (1) with directly observed moisture-content distributions will determine the validity of the basic assumption that the flow is directly related to the pressure P. It will also indicate the possibility of extrapolating both the pressure P and the aqueous conductivity C to cover the range below that possible experimentally.

The order of the calculated thickness of the water films constitutes the major objection to this conception. This point has been discussed by Houwink, who points out that at $pH = 5$ the average distance of the charge in the diffuse layer from the surface is about 10^3 A., and that, therefore, repulsion may reasonably be expected at distances of, say, 10^4 A. Freundlich (12) has worked out the fundamental idea in more detail, and his curves indicate that a stable mixture might be obtained with a separation of 5×10^3 A. It has been said that suspensions of a virus and of a bentonite of uniform particle size have been found to have a cubic quasi-crystal structure with a " lattice " of this order.

The plastic mixture may contain a comparatively large proportion of water. Moisture-contents of 40 and 50 per cent are not uncommon, and to account for this, an alternative explanation has been put forward by Terzaghi and others (13). In this, the clay particles are in contact at certain points, forming a series of bridges, or a honeycomb. In general the smaller particles, or the flatter, form the links between the larger, the spaces thus formed being filled with water. While this conception at first sight avoids the necessity for forces acting at comparatively large distances from the surface, more detailed consideration shows that this is not the case. The difficulties that arise with this postulate may be briefly reviewed as follows :—

1. Suspensions with as little as 10 per cent of clay are known to show rigidity under certain conditions. If a cubic arrangement of particles is assumed, this giving a large water-clay ratio, calculation shows that those particles must have a length and breadth at least 150 times their thickness. The clay mineral does not show such distinct cleavage.

2. The structure must therefore consist of chains of particles linked end to end. Any such chain must, for the rigidity of the whole, be capable of withstanding a stress in compression. It is therefore necessary to assume rigid linkage between particles, for which there is no explanation.

3. The structure must consist of bridges of such arbitrary number and strength that the logarithmic relation between P and M, described below, holds. This relation, however, is true for all clays, in which not only the grain size but the size distribution vary greatly.

4. During drying, it is to be expected that some, at least, of these bridges would be of sufficient strength to remain intact. The volumetric shrinkage, with its sharp cessation, is consequently very difficult to explain.

5. The original formation of such a structure in a mixture which has been thoroughly mixed and worked during its preparation involves not only attractive forces acting between certain parts of certain particles, but also repulsive forces between non-adjacent particles. These repulsive forces must be selective, and must act over much greater distances than 1000 A.

6. A reduction of particle size will reduce the scale of a honeycomb as a whole, and will not affect the clay-water ratio. Grinding, however, greatly increases the moisture-content at the same working consistency.

7. Systems such as mixtures of sand or flint with water, in which the particles are known to be in contact, do not show shrinkage or plasticity.

8. Excessively complicated inter-particle forces are required to account for plasticity. If the structure is capable of being destroyed by drying, it will be expected to break down under shear with a liberation of free water.

§ 2. THE PRESSURE : MOISTURE-CONTENT RELATION

The apparatus used for the determination of the pressure : moisture-content relation and of the aqueous conductivity, figure 2, is constructed entirely of nickel and follows the lines of a similar one used by Westman (14). It consists of a cylinder, closed at the bottom with a screw plug, and an accurately fitted piston. Both the piston and plug are doubly drilled, leading to the external system of interconnected

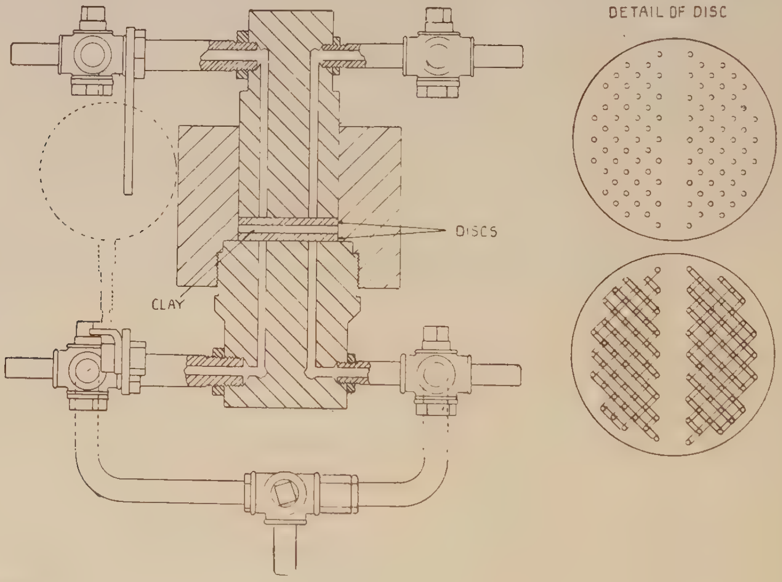

Figure 2. Apparatus used in determination of pressure : moisture-content relation and aqueous conductivity.

three-way taps. The clay sample, in the form of a disc $\frac{1}{8}$ in. thick, is covered top and bottom with a wet and fine-grained filter paper. Two drilled discs, with a series of channels on one side, serve to distribute the water flow uniformly over the cross-section. An Ames dial, reading in 1/10,000th in., indicates the movement of the piston relative to the cylinder.

The mechanical pressure on the piston is applied by means of a lever and weights, the whole apparatus being moved to various fixed distances from the fulcrum for varying loads. Ten such loads are normally used, covering the range 27 to 617 lbs.-in.$^{-2}$ pressure. The flow of water through the clay is obtained with a reservoir of adjustable height, and being very small, is measured by observing the position of the meniscus in a horizontal length of fine capillary tubing. The direction of flow through the clay is downward to avoid interference by air locks, although precautions are taken against these. The whole apparatus is situated in a room maintained thermostatically at a temperature of 25°c.

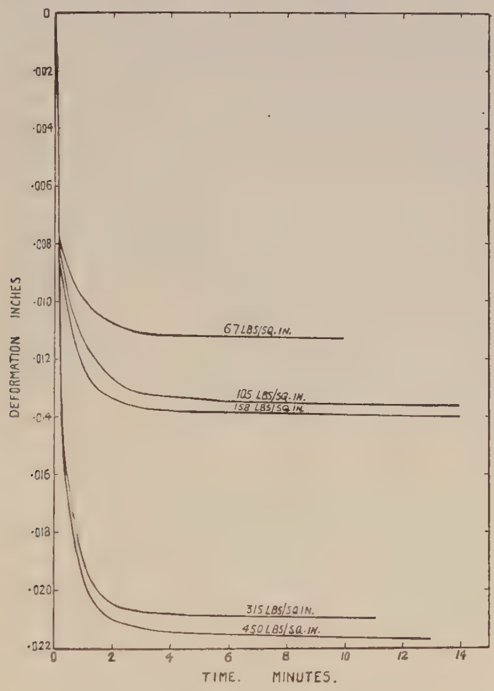

Figure 3. Typical time-thickness (moisture-content) curves.

The object of the double system of water channels was, following Westman, to remove air from the apparatus at the start of an experiment. He passed water under pressure in at the one side, across the face of the clay, and out at the other, but it was found that the water pressure required was very large, and if the flow was sufficient to remove the air, some of the clay was also carried over. A different technique has been used in which the sample, between its filter papers, is gently placed in position when the cylinder is already full of water. The slightly cloudy liquid which usually results is removed and replaced, and the upper nickel disc and piston forced down. There is thus little possibility of air locks. The water used is distilled and freshly boiled.

The pressure : moisture-content relation was found for any particular clay by applying loads for periods necessary for equilibrium to be reached, quickly removing the clay sample and determining its moisture-content by drying out at 110° c. The possibility of erroneous figures due to the return of the expressed water into the clay upon the removal of the pressure was explored by comparing the moisture-contents of two identical samples, one of which was removed in some 15 seconds, while the other was left for two hours after the removal of the pressure. The moisture-contents were 24·55 per cent and 24·70 per cent respectively. The very small increase is indicative of the very large hysteresis which exists in a reversal of this kind. The existence of such hysteresis is well recognised (13), and is reflected in the fact that uniform moisture-contents can only be obtained by storage for very long periods. Its effects do not enter greatly into considerations of drying, and have been avoided throughout by taking all changes in moisture-content from high to low.

After the application of the load, equilibrium is reached in a period rarely exceeding twenty minutes. Typical examples of the way in which the thickness of the sample changed with time are shown in figure 3. A variable part of the initial deformation in these curves is that necessary to cause the moulded disc to flatten and to fill the cylinder. A second variable lies in the exact manner of application of the load, which was by hand, and it was found that no conclusions could be reached from an examination of the times taken by various samples to reach equilibrium.

The relation between the applied pressure and the equilibrium moisture-content (figures 4, 8, 13, 18, 23, 28 and 33) is an exponential one, and may be expressed by $P=\alpha e^{-\beta M}$, where α and β are experimental constants for each clay. This empirical relation is essentially the same as that found by Terzaghi, by Hatch (13), and by others. Terzaghi, in particular, included a study of the reversibility of the process. Working with small pressures (below 42 lb.-in.$^{-2}$), he found that the pressure : moisture-content relation for these high moisture-contents was not reversible along the compression curve, the increase in moisture-content on release of pressure being less than the decrease over the corresponding pressure increase. He compared these pressure : moisture-content curves, or cycles, for different clays with those for mixtures of sand and mica, and, although the moisture-contents in the two cases were very different, concluded, from similarities in the shape of the curves, that the controlling factor was the proportion of flaky material present. This similarity should, however, be treated with caution, for the sand-mica mixture is not plastic and exhibits none of the other recognised properties of clay. It is possible that this is merely an interesting coincidence between two essentially dissimilar systems. As already indicated, the process of drying is concerned almost entirely with the single decrease in moisture-content, and the reversibility of the process and the hysteresis connected with it have not received detailed attention here. Complete reversibility, however, is not to be expected.

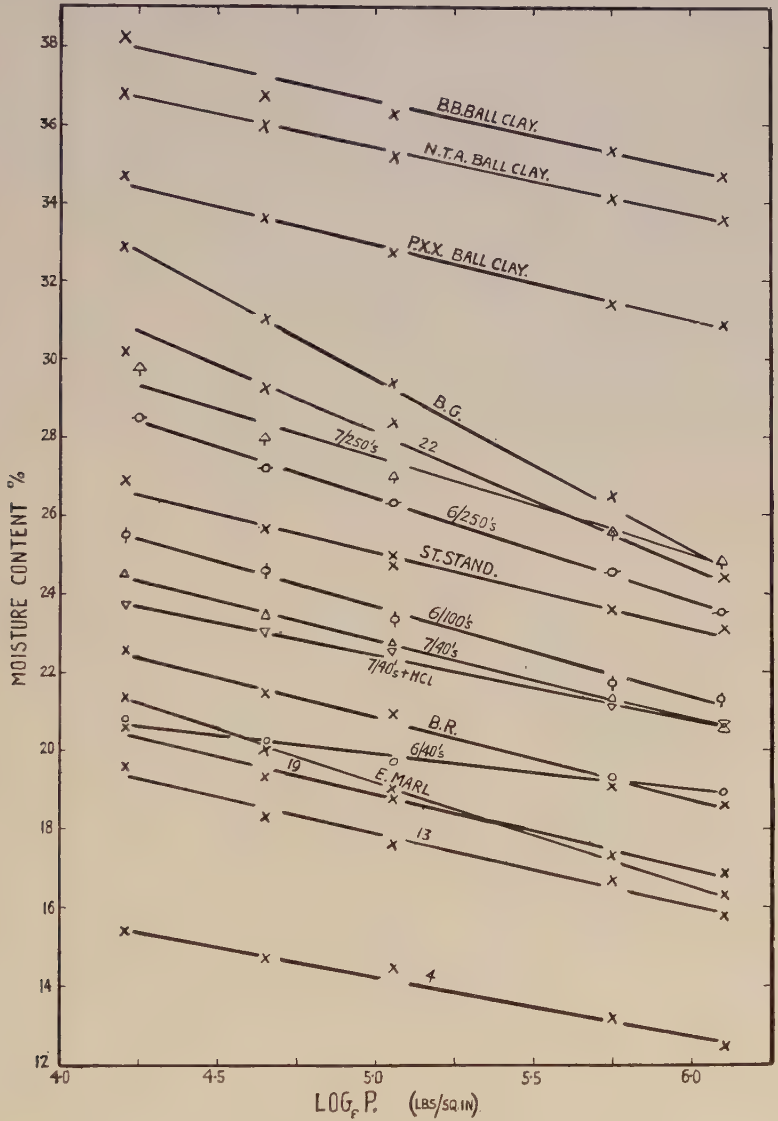

Figure 4. Relation between pressure and equilibrium moisture-content.

The process of application of pressure is accompanied by plastic deformation and its associated shear hardening, in which new and stable arrangements of particles are found. These particles will not return to their original positions upon the removal of the pressure.

Table 1 gives the values of the constants α and β for different clays arranged in ascending order of β. The abbreviation /40 shows the size of lawn through which the clay was passed in grinding.

Table 1. Values of α, β, θ, and ϕ in the expressions
$P=\alpha e^{-\beta M}$ and $C=\theta e^{\phi M}$

Reference	Origin	α	β	θ	ϕ
Bentonite	Unknown	$3{\cdot}03\times10^{8}$	0·053	—	—
	Fireclays and brickclays				
32B	Bedfordshire brickclay	$2{\cdot}76\times10^{10}$	0·266	$1{\cdot}58\times10^{-16}$	0·281
B.G.	Belgian fireclay	$1{\cdot}24\times10^{10}$	0·282	—	—
22/40	Bedfordshire brickclay	$4{\cdot}20\times10^{10}$	0·296	—	—
21/40	,, ,,	$5{\cdot}23\times10^{10}$	0·304	—	—
31	,, ,,	$1{\cdot}74\times10^{11}$	0·331	$1{\cdot}42\times10^{-15}$	0·210
Etruria marl	North Staffs	$8{\cdot}91\times10^{9}$	0·355	—	—
26	Somerset brickclay	$1{\cdot}85\times10^{11}$	0·374	a $3{\cdot}68\times10^{-16}$	0·315
				b $2{\cdot}45\times10^{-16}$	0·323
6/250	Stourbridge fireclay	$4{\cdot}11\times10^{11}$	0·402	—	—
7/250	,, ,,	$1{\cdot}01\times10^{12}$	0·420	—	—
6/100		$2{\cdot}92\times10^{11}$	0·445	—	—
B.R.	Belgian fireclay	$2{\cdot}12\times10^{11}$	0·483	—	—
7/40		$7{\cdot}86\times10^{11}$	0·493	—	—
7/8		$7{\cdot}34\times10^{11}$	0·512	a $2{\cdot}61\times10^{-17}$	0·500
				b $2{\cdot}14\times10^{-18}$	0·639
13/40	Leeds fireclay	$1{\cdot}10\times10^{11}$	0·521	—	—
St. St.	Stourbridge fireclay washed and blunged	$1{\cdot}03\times10^{13}$	0·552	$1{\cdot}12\times10^{-17}$	0·506
19/40	S. Lancs brickclay	$3{\cdot}79\times10^{11}$	0·559	—	—
7/40+0·25 HCl		$3{\cdot}46\times10^{12}$	0·565	—	—
28	Stourbridge fireclay	$3{\cdot}71\times10^{12}$	0·565	$3{\cdot}03\times10^{-17}$	0·463
3	Stourbridge fireclay	$9{\cdot}15\times10^{11}$	0·570	a $2{\cdot}02\times10^{-17}$	0·583
				b $3{\cdot}15\times10^{-17}$	0·553
4/40	Scotch fireclay	$1{\cdot}05\times10^{11}$	0·645	—	—
27	Stourbridge fireclay	$2{\cdot}08\times10^{11}$	0·660	$3{\cdot}23\times10^{-16}$	0·578
B	N. Staffs fireclay	$3{\cdot}42\times10^{12}$	0·673	—	—
30D	Leeds fireclay	$3{\cdot}24\times10^{11}$	0·699	$8{\cdot}96\times10^{-17}$	0·584
29	Stourbridge fireclay	$8{\cdot}25\times10^{12}$	0·706	$9{\cdot}12\times10^{-18}$	0·650
30C		$3{\cdot}72\times10^{11}$	0·720	$1{\cdot}08\times10^{-16}$	0·608
30B		$3{\cdot}46\times10^{11}$	0·728	$6{\cdot}45\times10^{-17}$	0·652
30A		$5{\cdot}49\times10^{11}$	0·783	$1{\cdot}74\times10^{-17}$	0·780
		Ball clays			
P.X.X.	Commercial clay	$2{\cdot}44\times10^{14}$	0·516	—	—
34	,, ,,	$1{\cdot}12\times10^{15}$	0·552	$6{\cdot}79\times10^{-19}$	0·459
N.T.A.	,, ,,	$4{\cdot}77\times10^{15}$	0·565	—	—
B.B.	,, ,,	$5{\cdot}58\times10^{16}$	0·614	—	—
33	,, ,,	$3{\cdot}66\times10^{17}$	0·650	$5{\cdot}86\times10^{-20}$	0·475
		China clays			
"Colloidal"	Finest fraction	$3{\cdot}08\times10^{11}$	0·227	—	—
36	Commercial clay	$4{\cdot}12\times10^{13}$	0·433	$3{\cdot}15\times10^{-18}$	0·480

N.B.—Repeats are on separate batches.

The brickclays and fireclays may be considered as similar in nature, differing mainly in iron content. Let it be assumed that the repulsion between opposing particle surfaces is of the form $P = ae^{-bx}$, where x is the distance between them, and the same per unit surface area throughout this class of clay. The clay particles are best considered as discs, but the argument is similar and is considerably simplified by taking them to be cubes of side l. The water associated with a single cube is, to the first approximation, $3l^2x$. The surface factor J (surface area per unit weight of clay) is $J = 6l^2/l^3\rho = 6/l\rho$, where ρ is the density of the solid and the moisture-content $M = (3l^2x/l^3\rho)100 = 50Jx$.

Hence $P = ae^{-bM/50A}$, which is in agreement with the experimental data, for as α is equivalent to a, it should be the same for all the clays in this class, which is roughly true. Secondly, β is inversely proportional to the surface factor J. Reference to Table 1 shows that the clays arranged in ascending order of β are also in descending order of surface factor. At the one end of the scale are the finely divided Bedfordshire clays and the Etruria marl. At the other come much larger-grained and poorly plastic fireclays. The difference between the finest fraction china clay and the commercial variety is marked, as is the value of β for the bentonite. The effect of grinding through different sizes of lawn is in further agreement.

To the second approximation

$$P = ae^{-bM/50A} \times e^{bM^2/15000A}.$$

The additional term is very small and appears as a small increase in the value of α with increase of β.

§ 3. THE AQUEOUS CONDUCTIVITY

The variation with moisture-content of the aqueous conductivity was determined by successively increasing the applied pressure on the same clay sample, the moisture-contents corresponding to these pressures being known. Preliminary experiments showed (*a*) that under pressure heads of the order of two feet of mercury, the rate of flow was not constant, but decreased slowly with time; with pressure heads of up to three feet of water, the rate of flow was constant and proportional to the pressure head applied, and (*b*) that over the range of temperature available, 15° C., the rate of flow varied as the normal viscosity of water.

The importance of working with comparatively large batches of clay, each considered distinct, is shown in figures 5 and 6. In figure 5, Expts. 150 and 151 are repeats on a single batch of Clay 7, but the clay for Expt. 152 was ground and mixed separately. The curves in figure 6 are for two separately mixed batches of Clay 3. Further data on other clays are given in figures 9, 14, 19, 24, 29 and 34. The aqueous conductivity may be represented by an exponential relation rather similar to that between pressure and moisture-content, $C = \theta e^{\phi M}$, θ and ϕ being again experimental constants for each batch of clay. Hatch gives a somewhat similar expression and points out that the relation between conductivity and moisture-content given by Terzaghi is not even approximately true. This empirical formula is subject to the reservation that there is, in general, a break in

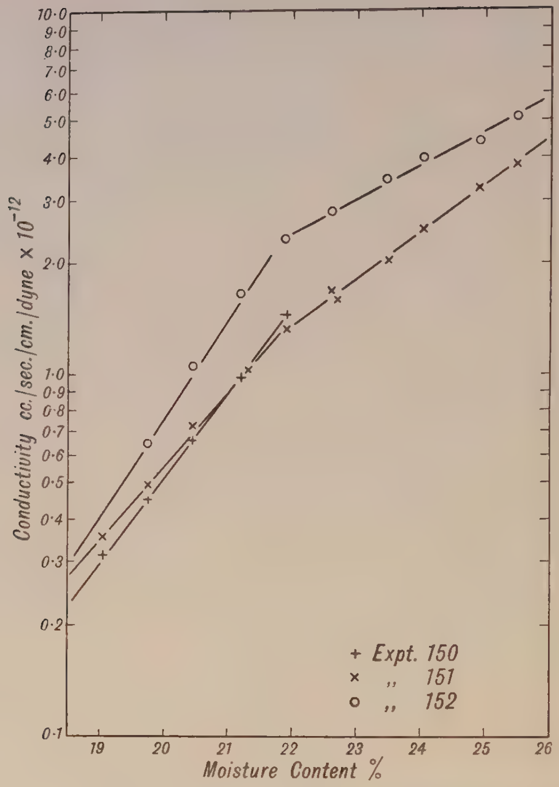

Figure 5. Aqueous conductivity of Clay 7, for two discrete batches.

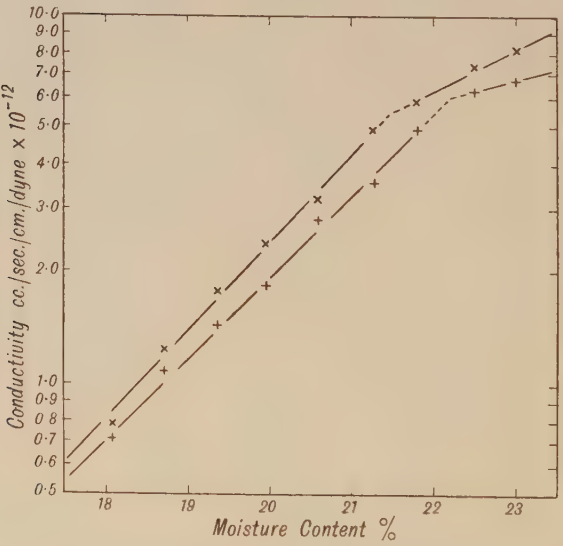

Figure 6. Aqueous conductivity of Clay 3, for two discrete batches.

the curve at some critical moisture-content, and two sets of values for θ and ϕ are necessary, valid respectively above and below the critical moisture-content. It will be noted that this critical moisture-content occurs roughly at the same value of the pressure P for the different clays, i.e., at about the same thickness of water film. We are mainly concerned with the thinnest films and moisture-contents below this point.

The aqueous conductivity of all the clays investigated is of a very small order, 10^{-12} c.c.-sec.$^{-1}$-cm.$^{-1}$-dyne^{-1}, and 10^{-10} for the china clay. The variation with moisture-content is very large, a change in the latter of some 5 or 6 per cent being sufficient to reduce the conductivity to about one-tenth of its original value.

Various other formulae have been put forward to express the flow of water through a porous material and have met with some success in the case of large-pored systems such as sands. Smith(15) has derived on theoretical grounds an expression for the flow of a liquid through an ideal uniform soil considered as consisting of spheres of uniform radius. This is a rather involved function of $(1-\epsilon)^{2\ 3}$, ϵ being the porosity, or volume of water in unit volume of the mix. Evaluation of this variable for Clay 7 gives 0·0525 and 0·0824, corresponding to observed conductivity values of 0·642 and 5·07. Over the experimental range the observed change is some ten times that given by the theoretical expression. Lack of agreement of this order is also found with the simple method of Easter(16), in which cylindrical capillaries are assumed and their number and radius calculated, both in its original form and when modified for flow through films rather than capillaries. Of other formulae, the best known is that of Carman(17), in which the flow varies as $\epsilon^3/(1-\epsilon)^2$. This expression has the values 0·062 and 0·120 at the same observed conductivities of 0·642 and 5·07. The agreement is slightly better than with the formula due to Smith. Clay 7, however, is one of which the working moisture-contents are not particularly high, and gives a comparatively large variation in ϵ over the experimental range. Where the moisture-contents are high, as, for example, with Clay 33, the lack of agreement is more pronounced, Carman's expression having the values of 0·372 and 0·516 at observed conductivities of 0·90 and 6·50. These expressions, apart from the modification of Easter's, are in essence based on conceptions similar to those of Terzaghi. It is apparent that they all fail entirely to account for the experimental data.

Theoretical expressions such as these must be roughly comparable in their range of variation with change of moisture-content, and they can only be made to vary more rapidly by the supposition of very thick fixed films of water on the solid surface, such as have been found necessary by Carman and have been proposed for other reasons by Longchambron and Zajtman(18). They are, in essence, based on the fourth power of the radius, and it is obvious that expressions showing a much more rapid variation can be obtained if that radius be reduced by the subtraction of a constant quantity. Such formulae may in this way be made to fit the observed values roughly over the experimental range of moisture-content. They result, however, in the conductivity becoming zero at some comparatively high moisture-content well above the leather-hard when the thickness of the films is that of the assumed fixed layer. This would mean that the clay could not dry or shrink below this moisture-content, apart from any theoretical objections to the existence of very thick bound water layers.

In table 1, above, are the values of θ and ϕ for a number of clays, and for the lower range of moisture-content. It will be seen that β and ϕ increase together and are approximately equal. The general decrease of θ with increase of β follows from the observed fact that over the same range of the pressure P the conductivities of the different clays are of the same order.

§ 4. THE FLOW OF OTHER LIQUIDS THROUGH CLAY

Measurements of the rate of flow of benzene, nitrobenzene and pyridine through clay have been attempted. The clay (No. 30) was ground and carefully dried at 110° c., being then mixed with the pure liquid *in vacuo*. Conductivity experiments were carried out in the usual manner.

These three liquids pass through clay with great rapidity, the conductivities being of the order of, and not less than, 10^{-6} or 10^{-7}, i.e., some 10^5 times the rate of flow of water through the same clay. These high rates of flow produced great experimental difficulties, and the experiments were abandoned on the grounds of the unsuitability of the apparatus, the resistance of which was some twenty times that of the clay-liquid sample. It was attempted to eliminate this resistance by the use of samples of different thicknesses, but without success. Flow of the same order was found through a layer of sillimanite grog which had been graded to pass through an 8-mesh sieve, and lie on one of 16 mesh.

There were some slight indications that the rate of flow of nitrobenzene compared with that of benzene was rather less than could be attributed to the difference in viscosity, but no emphasis can be placed on this point.

§ 5. MOISTURE-CONTENT DISTRIBUTION IN LINEAR FLOW

Experimental determinations of the moisture-content distributions in clay bars drying from one end only have previously been made by Troop and Wheeler(19). They enclosed their clay in glass tubes which had been split longitudinally. This method has the disadvantage that the clay shrinks away from the glass during drying, and evaporation takes place from an increased area of surface. The use of a thin elastic covering on bars of one inch cross-section and either six or seven inches long has avoided this difficulty. In the earlier experiments the bars were moulded and subsequently covered. Later, to avoid excessive handling, the bars were extruded from a small de-aired "stupid" directly into the covering. A similar method of covering has since been used by Morgan and Hursh(20), who, like Troop and Wheeler, do not attempt any explanation of the moisture distributions they obtained.

The small dryer, figure 7, constructed for the drying of these bars is situated in the same thermostatically controlled room as the other apparatus. It consists of a rectangular glass vessel divided longitudinally to within one and a half inches of its closed end by a metal partition. This vessel is clamped (with a rubber gasket) to a double metal elbow, one side of which contains a small fan and the other wet- and dry-bulb thermometers. To the lower side of this elbow is attached a second glass tank containing a solution, in most cases of sulphuric acid, of a vapour pressure suitable to the rate of drying required. The air contained in this dryer is continuously recirculated over this controlling liquid and over the four bars used in any one experiment, these being placed on glass plates, two on each side of the partition. The rate of drying of the bars is determined by weighing at intervals, while at suitable stages one of the bars is removed and cut into sections, $\frac{1}{4}$ in. in thickness and $\frac{1}{2}$ in. apart. The moisture-contents of these sections are found by the usual method of drying out at 110° c.

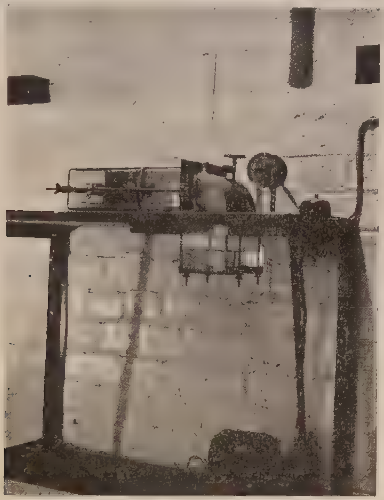

Figure 7. Dryer used for determination of moisture-content distributions.

The basic differential equation (1) may now be written

$$\alpha\beta\theta\frac{\partial}{\partial x}\left[e^{(\phi-\beta)M}\frac{\partial M}{\partial x}\right]=\frac{\partial}{\partial t}\left(\frac{M}{S+M}\right) \qquad \ldots\ldots(2),$$

which is incapable of direct solution.

Since, however, $(\phi-\beta)$ is small compared with either ϕ or β, we may make the approximation of writing

$$\alpha\beta\theta e^{(\phi-\beta)M_c}\frac{\partial^2 M}{\partial x^2}=\frac{\partial}{\partial t}\left(\frac{M}{S+M}\right) \qquad \ldots\ldots(3),$$

where M_c has a suitably chosen value. The assumption is that $\phi=\beta$, and the clay will have its true conductivity at the moisture-content M_c, which must therefore be the mean moisture-content to be associated with the change between the initial and final states.

To reduce (3) to the heat-flow equation

$$\frac{\partial^2 y}{\partial x^2}=\kappa\frac{\partial y}{\partial t} \qquad \ldots\ldots(4),$$

the assumption that $(S+M)$ is constant must also be made. This is the volume of the mixture containing 100 gm. of dry clay, and represents the shrinkage. This difficulty might at first sight be overcome by a transference of variable from M to ϵ, the porosity. However, in the drying of a bar, the final volume is not the same as the initial, and the total change in ϵ is not proportional to the volume lost, i.e., to the rate of drying and the time. For, if ϵ_1 and ϵ_2 are the initial and final states corresponding to M_1 and M_2,

$$\epsilon_1-\epsilon_2=\frac{M_1}{S+M_1}-\frac{M_2}{S+M_2}=\frac{(M_1-M_2)S}{(S+M_1)(S+M_2)}.$$

It is necessary that the integral of a solution of (3) shall be equal to the loss in weight of the bar, and the variable M must therefore be used. The term $(S+M)$ has been fixed at the initial moisture-content at which the dimensions of the bar are defined. It will be noted that, in general, ϕ is less than β, and the error involved in making both these assumptions is less than in either separately.

(3) may therefore be written

$$\alpha\beta\theta e^{(\phi-\beta)M_c}(S+M)\frac{\partial^2 M}{\partial x^2}=\frac{\partial M}{dt} \qquad \text{......(5).}$$

It is required to know the solution of (5) subject to the boundary conditions:

(*a*) The moisture-content initially is uniform, i.e., $M=M_0$ at $t=0$ for all values of x.

(*b*) The rate of drying at the surface is constant, i.e., $C\frac{\partial P}{\partial x}=R$ at the drying surface.

(*c*) At the non-drying end of the bar $\partial M/\partial x$ is zero for all values of time, i.e., the bar may be considered as one-half of a bar twice the length drying from both ends.

By suitable adjustment of the units of measurement of time, (5) can be written:

$$\frac{\partial^2 M}{\partial x^2}=\frac{\partial M}{\partial t} \qquad \text{......(6).}$$

Using Heaviside's operational method and writing $\frac{\partial}{\partial t}=\sigma=q^2$, (6) becomes

$$\frac{\partial^2 M}{\partial x^2}=q^2 M \qquad \text{......(7).}$$

A. *Solution in a form suitable for small values of t*

Taking the origin at the drying surface and denoting the length of the bar by l, the boundary conditions become:

$$\left[\frac{\partial M}{\partial x}\right]_{(x=0)}=A,$$

$$\left[\frac{\partial M}{\partial x}\right]_{(x=2l)}=-A.$$

A formal solution satisfying these conditions is:

$$M = M_0 - \frac{A}{q}\frac{\cosh q(x-l)}{\sinh ql} \qquad \ldots\ldots(8).$$

Hartree(21) has shown that the solutions of several one-dimensional problems in diffusion and heat conduction can be expressed conveniently in terms of the functions obtained by repeated integrals of the error-function. This has been defined by Jeffreys(22) as:

$$\operatorname{erf}(w) = \frac{2}{\sqrt{\pi}}\int_0^w e^{-\xi^2}\,d\xi\,.$$

To be consistent with this, Hartree has taken the error-function complement $\chi(w)$ to be the function:

$$\chi(w) = \frac{2}{\sqrt{\pi}}\int_w^\infty e^{-\xi^2}\,d\xi \qquad \ldots\ldots(9).$$

Repeated integrals of this function $\chi_n(w)$ are defined as:

$$\chi_n(w) = \int_w^\infty \chi_{n-1}(\xi)\,d\xi \qquad \ldots\ldots(10),$$

with $\chi_0(w) = \chi(w)$ as in (9).

The operators $q^{-n}e^{-qx}$ which occur in the Heaviside operational forms of the solutions can be shown(23) to have the following interpretations:

$$\sigma^{-m}e^{-qx} = (4t)^m \chi_{2m}\left(\frac{x}{2t^{\frac{1}{2}}}\right) \qquad \ldots\ldots(11)$$

and

$$q^{-n}e^{-qx} = (4t)^{\frac{1}{2}n} \chi_n\left(\frac{x}{2t^{\frac{1}{2}}}\right) \qquad \ldots\ldots(12).$$

In particular for $n=1$,

$$q^{-1}e^{-qx} = (4t)^{\frac{1}{2}} \chi_1\left(\frac{x}{2t^{\frac{1}{2}}}\right) \qquad \ldots\ldots(13).$$

Expanding (8) in negative exponentials,

$$\begin{aligned} M &= M_0 - \frac{A}{q}\frac{e^{q(x-l)} + e^{-q(x-l)}}{e^{ql} - e^{-ql}} \\ &= M_0 - \frac{A}{q}\left[e^{-qx} + e^{-q(2l-x)} + e^{-q(2l+x)} + e^{-q(4l-x)} + e^{-q(4l+x)} + \ldots\right]. \end{aligned}$$

Interpreting, using (13),

$$\begin{aligned} M = M_0 - 2At^{\frac{1}{2}}\Bigg[&\chi_1\left(\frac{x}{2t^{\frac{1}{2}}}\right) + \chi_1\left(\frac{2l-x}{2t^{\frac{1}{2}}}\right) + \chi_1\left(\frac{2l+x}{2t^{\frac{1}{2}}}\right) \\ &+ \chi_1\left(\frac{4l-x}{2t^{\frac{1}{2}}}\right) + \chi_1\left(\frac{4l+x}{2t^{\frac{1}{2}}}\right) + \ldots\Bigg] \qquad \ldots\ldots(14). \end{aligned}$$

B. *Solution in a form suitable for large values of t*

Transferring the origin to the non-drying end of the bar, (8) becomes

$$M = M_0 - \frac{A}{q}\frac{\cosh qx}{\sinh ql} \qquad \ldots\ldots(15).$$

If $\sigma = \frac{\partial}{\partial t}$, then (with suitable conditions on $F(\sigma)$ which are satisfied in this case) according to Jeffreys⁽²⁴⁾,

$$\frac{f(\sigma)}{F(\sigma)} \equiv \frac{f(0)}{F(0)} + \Sigma_\gamma \frac{f(\gamma)}{\gamma F'(\gamma)} e^{\gamma t},$$

where $\gamma = \sigma$ are roots of the equation $F(\sigma) = 0$.

Here $F(\sigma) = q \sinh ql$, and the roots of $F(\sigma) = 0$ are $q = 0$ and $\pm in\pi$, i.e.,

$$\sigma = 0 \qquad \ldots\ldots(16)$$

and

$$\sigma = -\frac{n^2\pi^2}{l^2} \qquad \ldots\ldots(17).$$

Taking (16),

$$\frac{1}{q}\cdot\frac{\cosh qx}{\sinh ql} \sim \frac{1 + \frac{1}{2}q^2x^2}{q^2 l(1 + \frac{1}{6}q^2l^2)}$$
$$\sim \frac{1}{l}\left(\frac{1}{q^2} + \frac{x^2}{2} - \frac{l^2}{6}\right)$$

for small values of q.

The interpretation of $\frac{1}{q^2}$ being t, the contribution of this root to M is

$$-\frac{A}{l}\left(t + \frac{x^2}{2} - \frac{l^2}{6}\right) \qquad \ldots\ldots(18).$$

Taking (17) we have

$$f(\gamma) = \cosh qx = \cosh \frac{in\pi x}{l} = \cos \frac{n\pi x}{l}$$

and

$$F(\sigma) = q \sinh ql$$
$$F'(\gamma) = \left(\frac{dF}{d\sigma}\right)_{\sigma=\gamma} = \frac{1}{2q}\left(\frac{dF}{dq}\right)_{q=in\pi/l}$$
$$= \frac{1}{2q}[ql\,(\cosh ql + \sinh ql)]_{q=in\pi/l}$$
$$= \frac{1}{2}\left(l \cosh in\pi + \frac{1}{q}.0\right)$$
$$= (-1)^n . \tfrac{1}{2}l.$$

Hence

$$\frac{f(\gamma)}{\gamma F'(\gamma)} = \frac{\cos\frac{n\pi x}{l}}{\left(\frac{-n^2\pi^2}{l^2}\right)(-1)^n . \frac{1}{2}l} = (-1)^{n-1} . \frac{2l}{n^2\pi^2} . \cos\frac{n\pi x}{l},$$

and the contribution of this root to M is

$$-\frac{A}{l}(-1)^{n-1}\frac{2l^2}{n^2\pi^2}\left(\cos\frac{n\pi x}{l}\right)e^{-n^2\pi^2t/l^2} \qquad \ldots\ldots(19).$$

Adding (18) and (19) we have

$$M=M_0-\frac{A}{l}\left[\left(t+\frac{x^2}{2}-\frac{l^2}{6}\right)-\frac{2l^2}{\pi^2}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2}\cos\frac{n\pi x}{l}\,.\,e^{-n^2\pi^2t/l^2}\right] \ldots\ldots(20).$$

For large values of t, this reduces to

$$M=M_0-\frac{A}{l}\left(t+\frac{x^2}{2}-\frac{l^2}{6}\right) \qquad \ldots\ldots(21).$$

Certain points in the application of these solutions may be commented upon. The value of M_c is the average moisture-content to which the expression $C=\theta e^{\phi M}$ has been applied, and is in general the mean of the initial and final average moisture-contents. Where the rate of drying is great, and the drying time small, the moisture-content at the non-drying end of the bar is unaffected, and its conductivity cannot be allowed to influence the shape of the moisture distribution curve. The value of M_c was in these cases taken at the centre of the area lying between the final moisture distribution and the initial straight line. Since the production of a number of clay bars of equal and known moisture-content is an impossibility, the initial moisture-content and volume were calculated for each bar from the integrated final moisture distribution curve and the initial and final weights. The cross-sectional area, difficult of measurement on a soft bar which must not be handled excessively, was obtained from the volume and length.

The value of t in the adjusted units determined which of the two solutions (14) or (21) should be used. The numerical value of $2\chi_1(x)$ ranges from 1·1284 at $x=0$ to 0·0008 at $x=2{\cdot}2$. Thus the series in (14) is highly convergent, and when $\frac{2l}{2t^{\frac{1}{2}}}>2{\cdot}2$ only the first two terms are necessary. When $\frac{2l}{2t^{\frac{1}{2}}}<2{\cdot}2$ the simple solution (21) is applicable.

§ 6. EXPERIMENTAL DATA

In the diagrams below, the smooth curve is that calculated, and the points are experimental. The curves are in each case obtained with the values of ϕ and θ relevant to the final moisture-content of the bar, and where the critical moisture-content occurs in the moisture distribution, curves calculated with both sets of values are given, that applicable to the higher moisture-contents being the broken line. Data on a number of different clays are necessary to show that the agreement is general, and not a property of any one particular clay.

Clay 28

This fireclay was ground to pass a 16-mesh sieve. Its pressure : moisture-content relation is shown in figure 8, and its conductivity in figure 9. In addition to those given in table 1, the properties of the clay are :—$S=46{\cdot}3$; $M_L=16{\cdot}25$ per cent; $V_D=61{\cdot}7$ c.c.; θ and ϕ (above 24·5 per cent) $=7{\cdot}82\times10^{15}$ and 0·233 respectively.

Comparisons of experimental and calculated distributions are given in figures 10 (in which the scale is stepped to separate the curves), 11 and 12, the average rates of drying of the four bars in each experiment being $0{\cdot}797 \times 10^{-6}$, $1{\cdot}85 \times 10^{-6}$ and $5{\cdot}46 \times 10^{-6}$ gm.-sec.$^{-1}$-cm.$^{-2}$ respectively.

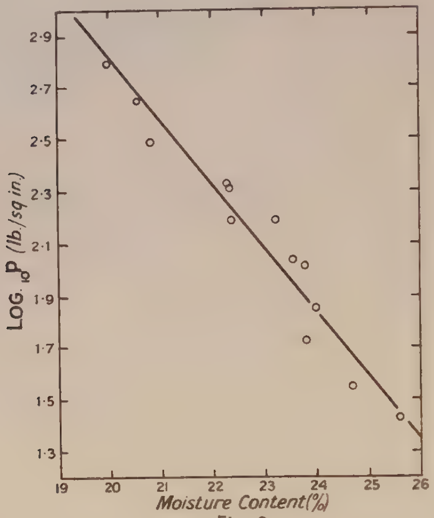

Fig. 8.

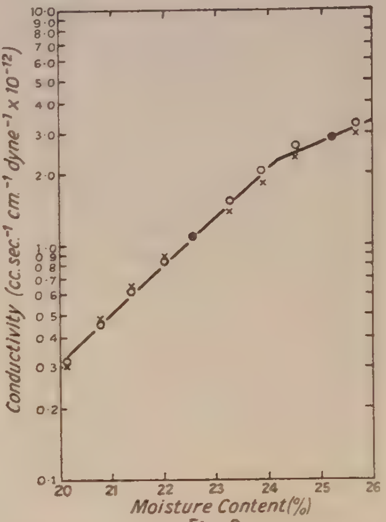

Fig. 9.

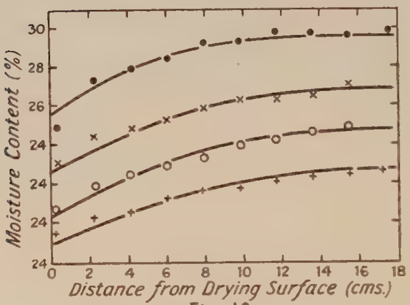

Fig. 10.

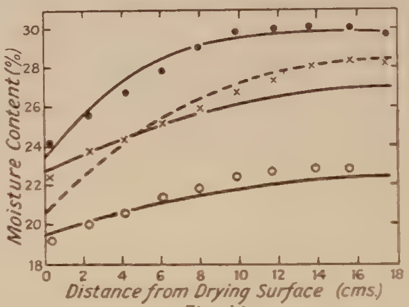

Fig. 11.

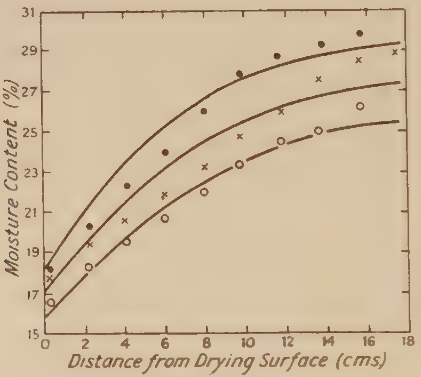

Fig. 12.

Figures 8, 9, 10, 11 and 12. Clay 28. Pressure : moisture-content relation, aqueous conductivity and comparison of theoretical and experimental moisture-content distributions.

Clay 29

This fireclay was ground to pass a 16-mesh sieve. Its pressure : moisture-content relation is shown in figure 13 and its conductivity in figure 14. In

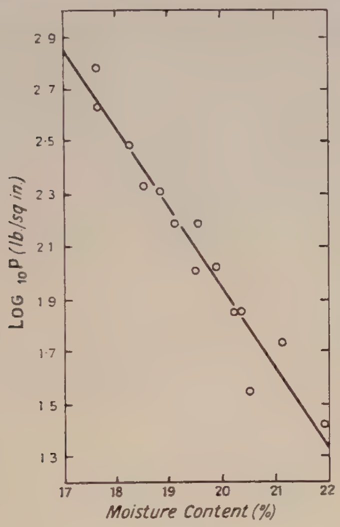

Fig. 13.

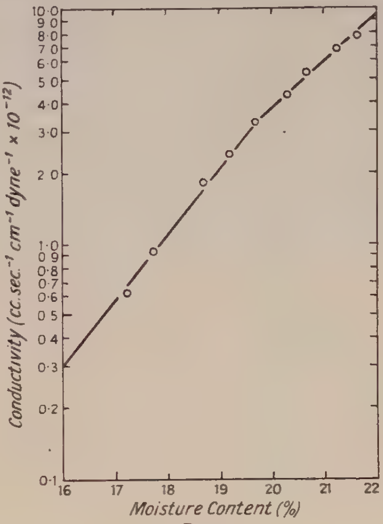

Fig. 14.

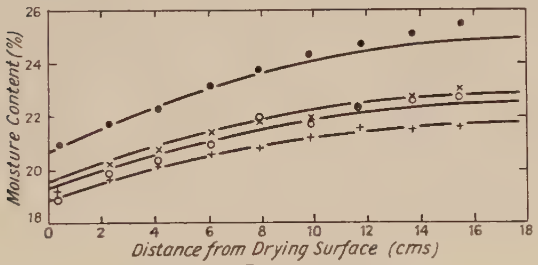

Fig. 15.

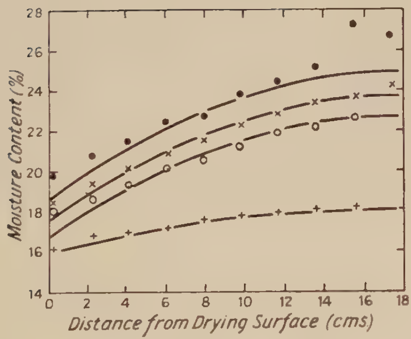

Fig. 16.

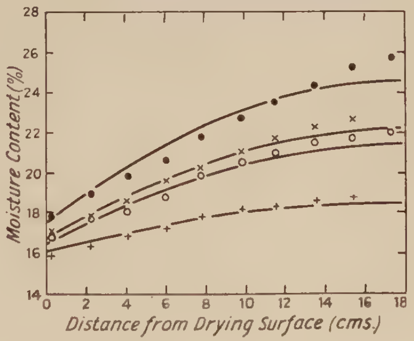

Fig. 17.

Figures 13, 14, 15, 16 and 17. Clay 29. Pressure : moisture-content relation, aqueous conductivity and comparison of theoretical and experimental moisture-content distributions.

addition to those given in table 1, the properties of the clay are:—$S=39{\cdot}4$; $M_L=13{\cdot}2$ per cent; $V_D=51{\cdot}7$ c.c.; θ and ϕ (above 19·7 per cent)$=4{\cdot}78\times10^{-16}$ and 0·448 respectively. Comparisons of experimental and calculated moisture

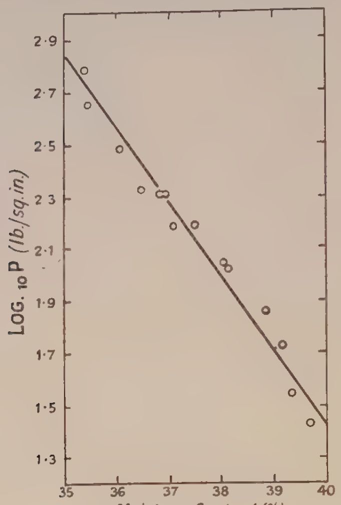

Fig. 18.

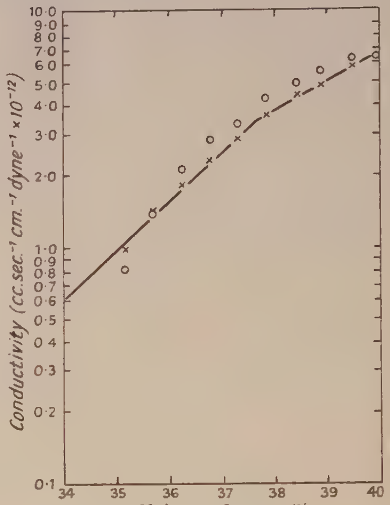

Fig. 19.

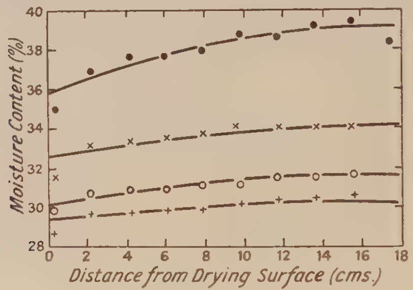

Fig. 20.

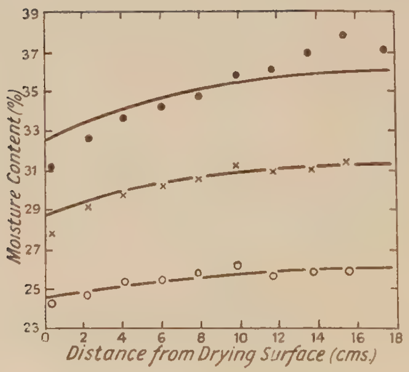

Fig. 21.

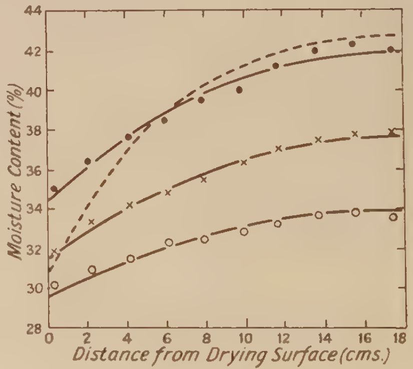

Fig. 22.

Figures 18, 19, 20, 21 and 22. Clay 33. Pressure : moisture-content relation, aqueous conductivity and comparison of theoretical and experimental moisture-content distributions.

distributions are given in figures 15, 16 and 17, the average rates of drying in these experiments being 2·12, 3·93 and $4{\cdot}25 \times 10^{-6}$ gm.-sec.$^{-1}$-cm.$^{-2}$ respectively.

Clay 33

This is a ball clay known commercially as B.B. and was broken down to pass a 16-mesh sieve. Its pressure : moisture-content relation is shown in figure 18, and its conductivity in figure 19. In addition to those given in table 1, the properties of this clay are :—$S=39{\cdot}5$; $M_L=25{\cdot}6$ per cent ; $V_D=62{\cdot}2$ c.c.; θ and ϕ (above 37·7 per cent) $=8{\cdot}73 \times 10^{-17}$ and 0·281 respectively. Comparisons of experimental and calculated distributions are given in figures 20, 21 and 22, the average rates of drying in these experiments being 3·50, 7·70 and $9{\cdot}90 \times 10^{-6}$ gm.-sec.$^{-1}$ cm.$^{-2}$ respectively.

Clay 34

This is a ball clay known commercially as P.X.X., and was broken down to pass a 16-mesh sieve. Its pressure : moisture-content relation is shown in figure 23, and its conductivity in figure 24. In addition to those given in table 1, the properties of this clay are :—$S=37{\cdot}8$; $M_L=20{\cdot}9$ per cent; $V_D=59{\cdot}4$ c.c.; θ and ϕ (above 33·8 per cent.) $=2{\cdot}93 \times 10^{-16}$ and 0·280 respectively. Comparisons of experimental and calculated distributions are given in figures 25, 26 and 27, the average rates of drying in these experiments being 9·58, 6·20 and $2{\cdot}80 \times 10^{-6}$ gm.-sec.$^{-1}$-cm.$^{-2}$ respectively.

Clay 36

This is a china clay known commercially as No. 100. Its pressure : moisture-content relation is shown in figure 28, and its aqueous conductivity in figure 29. It will be noticed that the aqueous conductivity of this clay is about one hundred times that of the other clays used, and is not accompanied by a change in the pressure : moisture-content relation of the same degree. At comparable rates of drying, differences in moisture content are therefore small. In addition to those given in table 1, the properties of this clay are :—$S=38{\cdot}0$; $M_L=28{\cdot}0$ per cent; $V_D=66{\cdot}7$ c.c.; θ and ϕ (above 35·3 per cent) $=3{\cdot}75 \times 10^{-14}$ and 0·214 respectively. Comparisons of experimental and calculated distributions are given in figures 30, 31 and 32, the average rates of drying in these experiments being 3·40, 11·0 and $19{\cdot}1 \times 10^{-6}$ gm.-sec.$^{-1}$-cm.$^{-2}$ respectively. The peculiar irregularity of the experimental points in the first curve of figure 30 is attributed to handling. The bars were picked up with the thumb and finger of both hands, and with this very high conductivity it is possible that the pressure thus applied was sufficient to cause a decrease in moisture-content at these points with a corresponding increase in the centre and ends of the bar. This handling was avoided in the other two experiments.

Although the properties of slips are outside the scope of this paper, it is interesting to consider the phenomenon of thixotropy in the light of the behaviour of this clay. The strength of a thixotropic system is due to a force network similar to that already explained, and equilibrium will exist when the particles are equidistant. The application of shear will produce inequalities of moisture-content, and if those portions of high moisture-content are liquid, aggregates of

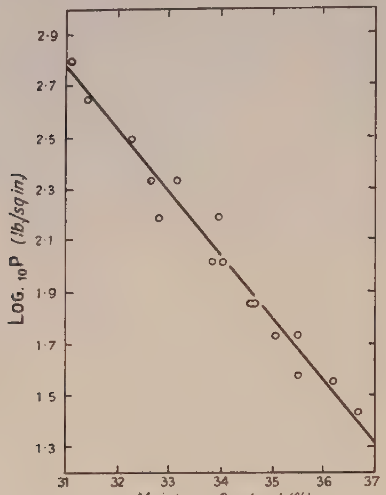

Fig. 23

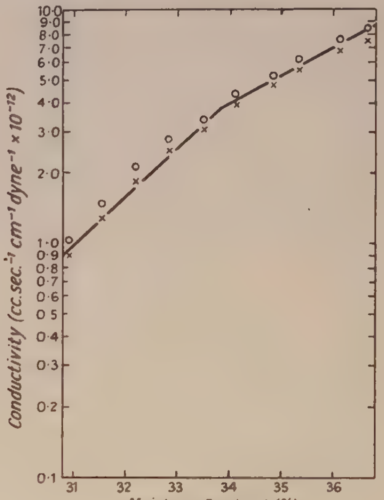

Fig. 24.

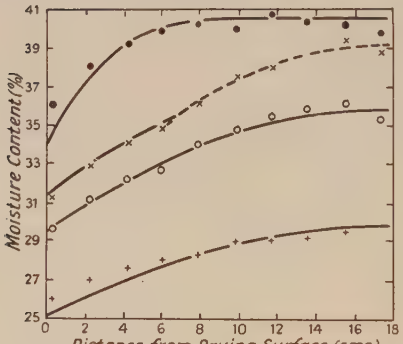

Fig. 25.

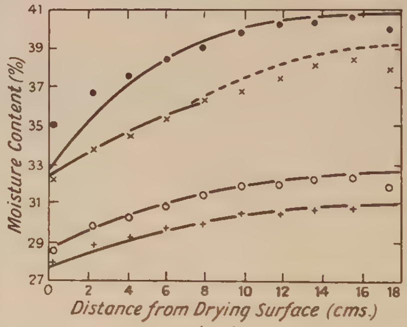

Fig. 26.

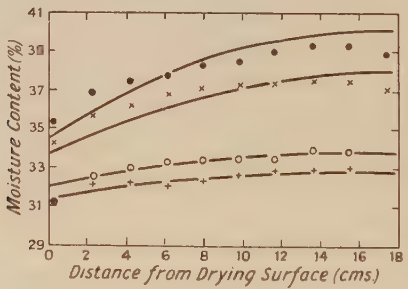

Fig. 27.

Figures 23, 24, 25, 26 and 27. Clay 34. Pressure : moisture-content relation, aqueous conductivity and comparison of theoretical and experimental moisture-content distributions.

lower moisture-content will flow freely over one another. On standing, uniform moisture-content and strength will again be attained, subject to the hysteresis effect mentioned earlier.

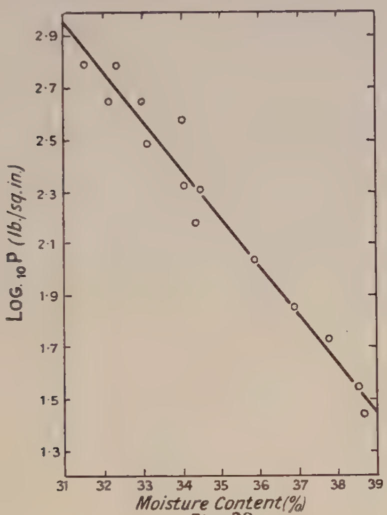

Fig. 28.

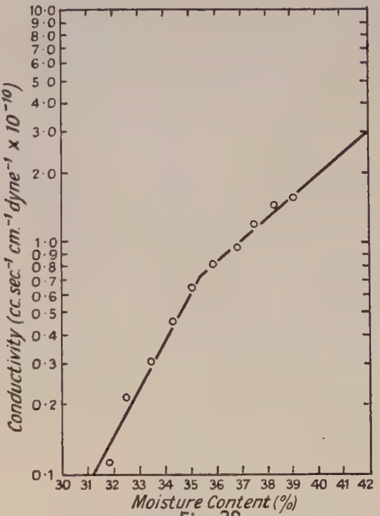

Fig. 29.

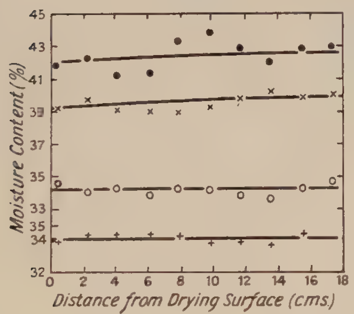

Fig. 30.

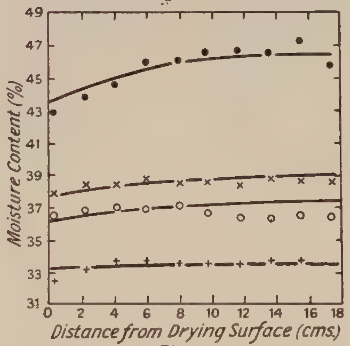

Fig. 31.

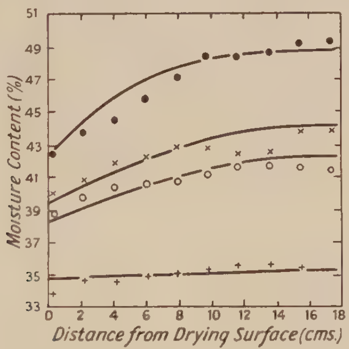

Fig. 32.

Figures 28, 29, 30, 31 and 32. Clay 36. Pressure : moisture-content relation, aqueous conductivity and comparison of theoretical and experimental moisture-content distributions.

Clay 30

The experiments on this fireclay (ground to pass an 8-mesh sieve) brought to light the large changes in aqueous conductivity which can be produced in a clay by drying and re-wetting and by mechanical working. The original batch of clay (A) having been exhausted, the dried scraps were covered with distilled water and allowed to break down into a slip. This was passed through an 8-mesh lawn and dried out until the clay was again plastic (B). A portion of this was worked between the fingers for ten minutes and stored for a short period (C), while subsequently the clay was again dried out and re-wetted (D). The pressure : moisture-content relations and aqueous conductivities of these batches are shown in figures 33 and 34, where it will be seen that the conductivity does not show the usual break. Drying out and re-wetting twice reduces the conductivity to about one-third of its original value. The decrease in β, table 1, follows the increase in surface factor produced by the treatment received.

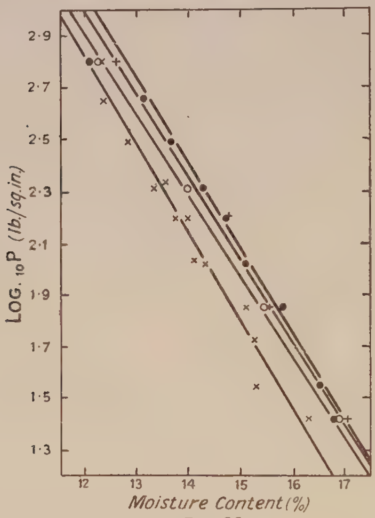

Fig. 33.

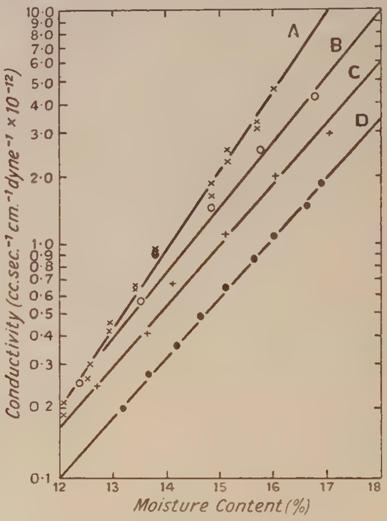

Fig. 34.

Figures 33 and 34. Clay 30. Pressure : moisture-content relation and aqueous conductivity.

Comparisons of experimental and theoretical moisture-content distributions on Batch A are given in figures 35, 36, 37 and 38. Figures 39 and 40 refer to a batch of clay similar to Batch C, but no great importance can be given to these, since the properties of the clay were not determined on the batch from which the bars were made. The broken curve in figure 39 was calculated with the constants of Batch A and shows the change in distribution brought about by the change in the properties of the clay.

The average rates of drying in the experiments of figures 35, 36, 37, 38, 39 and 40 were 1·00, 7·97, 7·75, 5·17, 3·70 and $3{\cdot}25 \times 10^{-6}$ gm.-sec.$^{-1}$-cm.$^{-2}$ respectively.

The shrinkage behaviour of this clay is $S=38{\cdot}6$; $M_L=11{\cdot}0$ per cent; $V_D=48{\cdot}25$ c.c.

Comments

The agreement between the experimental and theoretical moisture-content distributions appears to be amply good enough to justify the validity of the assumptions on which the latter were based. Furthermore, the agreement at low moisture-contents may be held to justify the extrapolation of the empirical expressions $P=\alpha e^{-\beta M}$ and $C=\theta e^{\phi M}$ to values of moisture-content below those at which direct measurement is possible.

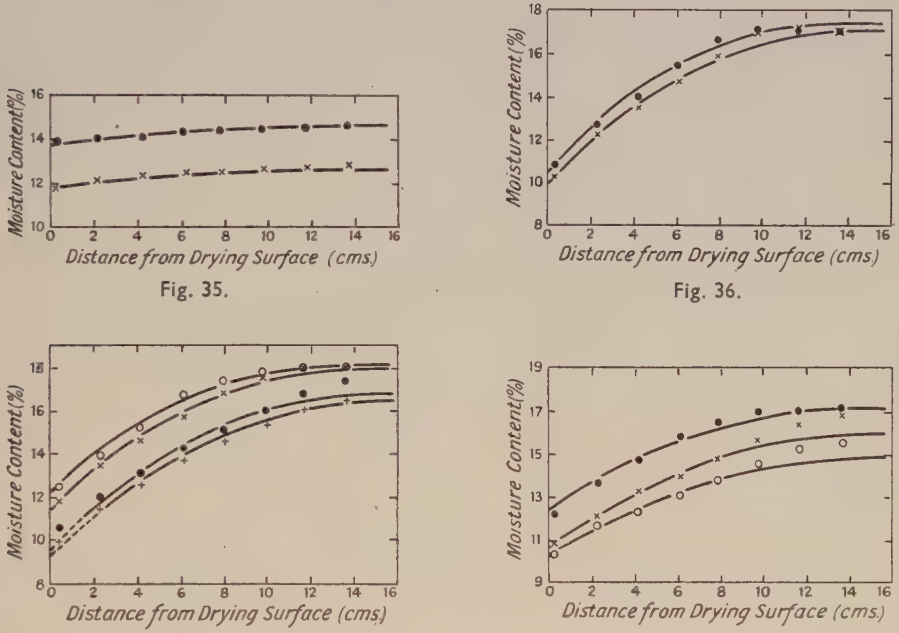

Fig. 35.

Fig. 36.

Fig. 37.

Fig. 38.

Figures 35, 36, 37 and 38. Clay 30 A. Comparison of experimental and theoretical moisture-content distributions.

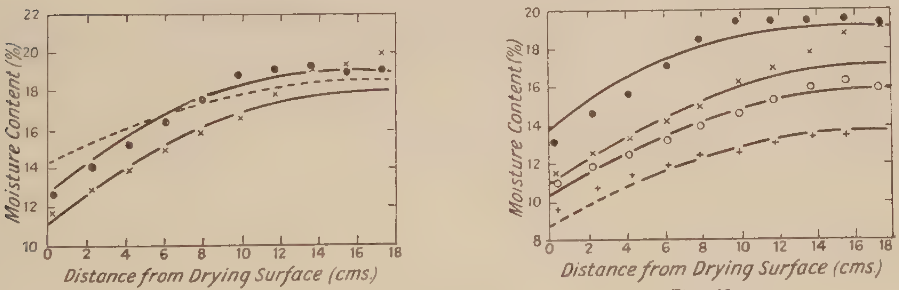

Fig. 39.

Fig. 40.

Figures 39 and 40. Clay 30 C. Comparison of experimental and theoretical moisture-content distributions.

§7. A POSSIBLE EXPLANATION OF THE OBSERVED AQUEOUS CONDUCTIVITY

It has been explained above that expressions for the flow of water through a porous body which are based on the normal laws of viscosity are incapable of explaining the experimental data. In particular, they either do not predict a variation as rapid as that of the observed conductivity, or they lead to the untenable conclusion that the conductivity becomes zero at some relatively high moisture-content. It is also obvious that there exists a very marked difference between the behaviour of water and that of other liquids.

The literature of the subject affords evidence both for and against abnormalities in the viscosity and other properties of water when in fine pores. Wilsdon, Bonnell and Nottage(25) note that experiments at the Building Research Station have shown that for several finely pored materials the hydrostatic suction necessary to remove water is of quite a different order from that which would be expected from the vapour-pressure relation. Similar anomalous behaviour was found when the same moist materials were brought into equilibrium with a solution of sugar of known osmotic pressure through a semi-permeable membrane. They proceed to mention other cases of anomalous behaviour which have been reported: e.g. Szigetti's measurements of vapour pressures and negative absorptions from sugar solutions(26). Glixelli and Wiertelak(27) found low values for the electrokinetic potential in silica gels which became normal in ignited gels. White, Urban and van Atta(28) found very low values for stream potentials in pyrex glass capillaries of less than 0·001 cm. diameter.

Shereshefsky(29) has produced evidence that the measured vapour pressure in small capillaries is much smaller than that calculated from the Kelvin equation. In particular, Wolkowa(30), in measuring the velocity of penetration of liquids into finely divided materials, found regular behaviour with non-polar liquids, but anomalous results with water and other polar liquids. Wilsdon, Bonnell and Nottage suggest that these anomalies may be due to the presence of chains of oriented water molecules, work being required to destroy them.

Bull and Wronski(31) have measured the viscosities of the first seven aliphatic alcohols through small capillaries in diaphragms of cellulose, carbon and glass. They found that in general the rate of flow through a given diaphragm appears to be a function both of the viscosity and of the degree of attraction between the liquid and the diaphragm, and make an estimate of the radius below which anomalous rates of flow occur.

Derjaguin(32), from measurements on the resistance offered by water to the movement of an oscillating lens, concluded that a water film $0{\cdot}1\,\mu$ thick has a rigidity of about 1/300 of that of lead. On the other hand, Bulkley(33), with the definite object of attempting to detect any departure from the laws of simple viscous flow, measured the rate of flow of various liquids through fine glass and platinum capillaries. The finest of these had inside diameters of $9{\cdot}35\,\mu$ and $5{\cdot}59\,\mu$, but he found that the rates were as expected. He concludes that the thickness of any immobile adsorbed layer on the walls of the capillaries could not have been in excess of two to three hundredths of a micron.

In reply to Wilsdon, Bonnell and Nottage, Bowden and Bastow(34) report measurements of the rate of flow of water between parallel plates, presumably of glass. The distance between these plates was 16,000 A., and they conclude that the viscosity of the film was the same as that of the liquid in bulk.

It is well established that the viscosity of some liquids, including water, is affected by the presence of an electric field. Alcock(35) found no effect with non-polar liquids, but increases up to 200 per cent in the viscosities of polar liquids, depending on the polar moments. Sokolov and Sosinskij(36), Bjornstahl(37), and Hertzog, Kudar and Paersch(38) have also recorded increases in viscosity, but Dobinski(39) attributes these effects to impurities in the liquids, as he found that the increase in viscosity decreased with progressive purification. Andrade and Dodd(40) are in agreement on this point.

The pressure : moisture-content relation indicates that there exists a repulsive force between the particle surfaces of the form $P=ae^{-bx}$. This is similar to Debye's expression(41) $\psi=Be^{-Kx}/x$ for the potential at a point distant x from the surface, in which the influence of the denominator is small compared with the exponential term. Freundlich and also Hamaker(42) have studied the interaction of two opposing surfaces. From their curves a repulsion of an exponential type may be expected over the range of distances, i.e., of moisture-contents, with which we are concerned here.

Immediately adjacent to the solid surface there is a layer of practically completely oriented water molecules. This may alternatively be considered as a strong crystal structure(43), for which there is evidence in the work of Bradley, Grim and Clark on bentonite(44). We may therefore define an intensity of orientation, or intensity of structure, I, as the probability of finding, at a given point, a water molecule oriented in a direction at right angles to the solid surface. At that surface the intensity of structure will be unity, but will be destroyed in subsequent layers by thermal agitation and the presence of positive ions. The decrease in strength of a plastic clay with rise in temperature may be mentioned as relevant here. If, as is reasonable to expect, the rate of destruction dI/dx is proportional to the intensity of structure at any point, I will be an exponential function of the distance from the surface, and may be written $I=e^{-tx}$, t containing the absolute temperature and the concentration of ions in the diffuse layer. The x-ray studies of liquids at interfaces by Trillat(45) are in agreement with this conception.

Further, the probability of finding a water molecule oriented in a direction at right angles to the surface is the probability of finding a polar moment so arranged. Thus the intensity of orientation may be considered to be intimately related to the rate of change of the potential. The repulsive force between two opposing surfaces is either directly connected with the potential at the point half-way between them, or, equivalently, arises from the mutual destruction of two equal and opposite intensities of structure at that point. In either case if $2d$ be the distance between the surfaces, we may put $P=\alpha e^{-\beta M}=\text{constant}\times e^{-td}$ or $\beta M=td$.

Following Andrade⁽⁴⁶⁾, viscosity is due to the communication of momentum from layer to layer, as in Maxwell's theory, but by temporary union at the periphery of the molecules in adjacent layers. This sharing of energy only takes place when the orientation is favourable, and the viscosity is roughly proportional to the probability that two molecules will combine. This probability of combination must be directly related to the probability of finding a molecule oriented at right angles to the shear, and the viscosity in a plane parallel to the clay surface will be proportional to the intensity of structure in that plane. For small values of x, where this effect is large compared with the normal energy sharing, we may neglect the normal viscosity and assume an anomalous viscosity $H=De^{-tx}$, D being influenced by thermal agitation, causing H to vary with temperature in the same way as the normal viscosity. It will be noted that this conception avoids that of a thick fixed film, and, in particular, does not require the sudden transition, within molecular distances, from the fixed adsorbed layer to free water.

An expression may now be found for the flow of water between parallel planes of length l and width s separated by a distance $2d$, and under a pressure head p. The tangential stress on the elementary plane is

$$F=2sDe^{-tx}\,dV/dx=f(x).$$

Hence

$$(p/l)\,2s\,dx+dx\,.\,f'(x)=0,$$

and since $dV/dx=0$ when $x=d$, the total flow between the planes is

$$R=\frac{2sp}{lDt^3}[2e^{td}-2td-2-t^2d^2].$$

But $td=\beta M$, and therefore

$$R=\frac{2sp}{lDt^3}[2e^{\beta M}-2\beta M-2-\beta^2M^2].$$

Over the range of experiment, βM is of the order of 10, and the last three terms amount to about 0·5 per cent of $2e^{\beta M}$. If they are neglected we may write

$$R=\frac{4sp}{lDt^3}e^{\beta M}.$$

Experimentally, $C=\theta e^{\phi M}$, and thus the rough equality between β and ϕ, pointed out above, may be explained. The influence of the terms $2\beta M+2+\beta^2M^2$ is to make the flow vary less rapidly than $e^{\beta M}$. Thus ϕ should be rather less than β, which is true for the majority of the clays investigated.

This conception will therefore account both for the pressure : moisture-content relation and for the very large variation in the aqueous conductivity with change of moisture-content, both of which are otherwise inexplicable. It cannot be denied that, as outlined above, certain factors of importance, notably the grain-size distribution, have of necessity been neglected. The large conductivity of the china clay 36, for example, may be attributed to a lack of the very fine particles which in other clays produce denser packing.

In conclusion it may be remarked that the property of plasticity gives strong indication that there exists some relationship between clay and water which is not to be found elsewhere. The material is, however, from the experimental point of view, a difficult one, since it will not lend itself to intimate observation, and complicated factors are at work which render any general behaviour throughout a number of different types quite uncommon. Indications of agreement are consequently of sufficient importance to warrant the drawing of conclusions, even though these may be of a somewhat tentative character.

§ 8. ACKNOWLEDGEMENTS

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THE ACTION OF ELECTRONS AND X RAYS ON PHOTOGRAPHIC EMULSIONS

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ABSTRACT. The measurement of electron densities by means of their photographic effect is discussed, and the results of many past investigations on the photographic action of light photons, x-ray photons and electrons are summarized. Attention is drawn to many points of similarity between high-velocity electrons and x-ray photons, and between low-velocity electrons and light photons. The effect of a gradual increase in the energy of the incident particle on the shape of the blackening curve is also discussed.

A theory based on an exponential decrease in the effect of the electrons due to absorption in the emulsion is advanced. The resultant formula is found to depend in a simple fashion on the grain properties of the emulsion, and on the absorption of the beam within it. An extension to variation with grain size is given, and the effect of varying absorption on the shape of the blackening curve is investigated. Previous formulae are compared with the new one, many approximations being found to be similar in nature.

Comparison of the formula with experimental results obtained by previous investigators and by the writer allows many of the grain properties of the emulsion to be correctly predicted, as well as the shape of the blackening curve, the range in the emulsion, etc. In particular, the apparent independence of the shape of the characteristic curve for x rays of the wave-length is deduced, and it proves possible to account for observed variations.

The quantitative agreement obtained in many cases tends to show the general correctness of the theory.

Some possible uses of the theory are appended.

§ 1. INTRODUCTION

In the study of a substance by electron or x-ray diffraction, the variation of diffracted beam intensity with direction is of primary importance. While the use of the ionization chamber and of the photographic plate has made the measurement of x-ray diffraction intensities a standard process, the corresponding use of the Faraday cylinder or of the photographic plate for the accurate measurement of diffracted electron-beam intensities is by no means so widespread. The Faraday cylinder is generally unsuited for measurement of the more complex electron-diffraction patterns. The photographic plate, moreover, is more sensitive, has greater resolving power and is less liable to error due to fluctuations of the incident beam. It is far more frequently used in electron-diffraction work, but unfortunately the relation between the electron density incident on a region of the plate and the corresponding blackening is not very simple. In most cases it has proved sufficient to estimate the intensity visually, since the blackening increases with electron density over a wide range, and this rough approximation may lead to very valuable information on certain aspects of crystal structure. Nevertheless, much information contained in the electron-diffraction pattern can only be made available by accurate measurements of the diffracted-beam intensities.

The relation between incident electron intensity and photographic blackening depends on many factors, and is generally determined experimentally under the conditions of the experiment. In the course of work of this type, the author has deduced a simple theoretical formula which represents the observed relation very satisfactorily. This formula applies not only in electron diffraction, but also in other branches of electronics, and for the measurement of minute currents. In many respects the photographic action of x rays is similar to that of high-energy electrons, and the same formula is found to apply. A comparison of the photographic action of high- and low-energy electrons and photons is also included.

§ 2. HISTORICAL

Many of the problems which arise in the study of the photographic action of x rays or electrons also occur when dealing with light rays, and in view of the many thorough investigations which have been carried out for this form of radiation it is of value to review some of the results obtained.

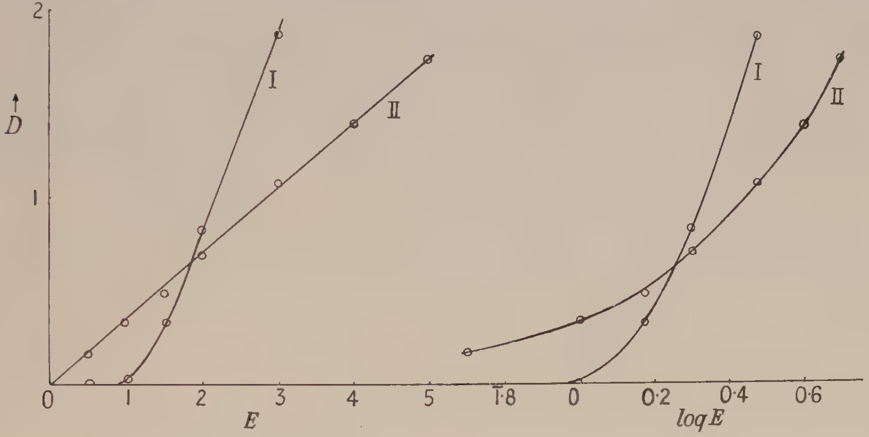
The photographic density D is defined by means of the equation $D = \log I_0 - \log I_E$, where I_0 and I_E are the relative intensities of light transmitted through the developed plate after no exposure and exposure of density E respectively. Common logarithms are here implied, Naperian logarithms being denoted as $\log_e N$.

If the radiation of density E is due to uniform radiation of intensity I over a period of t seconds, we have that $E = It$. The question immediately arises, does the observed blackening D depend on the total incident radiation E only, or does it depend on the separate values of I and t, i.e. on the time during which the constant exposure E has been made? The reciprocity or Bunsen-Riscoe law states that D is a function of E only, so that $D = D(E) = D(It)$. For light the reciprocity law is not valid, but a generalization due to Schwarzschild (1) is more satisfactory. This may be written

$$D = D(It^p),$$

where the value of p defines the degree of departure from the reciprocity law. Thus if $p = 1$, the law is valid. For light, values of p have been found ranging from 0·04 to 1·85, depending on the time of exposure.

The failure of the reciprocity law appears to indicate that some form of interaction occurs between different photons, or between their effects on the photographic grains. Although some of the theories (2) advanced to explain the photographic action of light take explicit account of this interaction, most of them consider the failure of the reciprocity law to be of secondary importance, and for simplicity assume its validity. It is interesting to note that at low temperatures, when the interaction may be deemed to become smaller, the reciprocity law becomes valid.



1 *a*: plot of *D* vs. *E*. 1 *b*: plot of *D* vs. log *E*.

Figures 1 *a* and 1 *b*. Characteristic curves for light (I) and x rays (II).

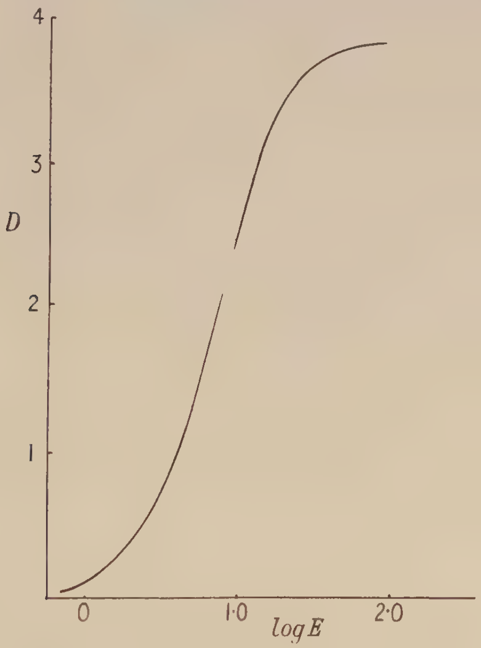


Figure 2. Characteristic curves for light, showing regions of under-exposure, normal exposure and over-exposure.

The relation found to exist for light between the photographic density and the incident radiation *E* is shown in figure 1 *a*, while the more usual plot relating *D* to log *E* is shown in figures 1 *b* and 2. There is an initial region of under-exposure where E increases without any appreciable increase in D. This is followed by the region of normal exposure when D and $\log E$ are related linearly. Finally, in the region of over-exposure, the rate of increase of D diminishes, so that finally it may decrease with increasing values of E (solarization). The intercept on the $\log E$ axis made by the straight portion of the curve is known as the *inertia* and defines the speed of the film in the H and D system.

Many attempts have been made to explain the observed form of the blackening curve, some early attempts being summarized by Ross (3). A later formula put forward by Silberstein (4) and based on the quantum theory leads to the expression

$$K = N(1 - e^{-na\epsilon}).$$

K is the number of photographic grains out of a total number N, which are rendered developable by an incident radiation consisting of n photons per unit area, a is the cross-sectional area of each grain, and ϵ is the probability of a grain becoming developable when struck by a photon. This formula was found to be fairly satisfactory in practice and led to a very small value of ϵ ($1{\cdot}2 \times 10^{-5}$). Unfortunately, the theory had to be abandoned in view of its inability to explain the failure of the reciprocity law, and more particularly since it predicted that D and E should be proportionate to one another at low intensities. This is in direct contradiction to experimental results, which, as mentioned above, indicate that only for radiation densities E above a certain value does D increase with E.

Various extension of the above and other theories, due to Silberstein (4), Toy, Sheppard, Trivelli, Loveland (5) and others, have failed owing to similar difficulties. It now appears necessary to assume that several photons are needed to render a grain developable, low values of the order of three or four photons per grain being generally found. Webb (6) has criticized these results on the grounds that they assume constant sensitivity of the various grains, determined by the minimum number of quanta necessary to render them developable, whereas this number is more likely to vary from one grain to another.

Busé (7) put forward an empirical formula which represents the observed relation between D and E over a given range very satisfactorily. It can be written

$$D = D_0 \log(t/T + 1),$$

D_0 and T being constants. This leads to an approximately linear relation between D and $\log t$ for sufficiently long exposures, while for short exposures D and t are proportional. If the reciprocity law is assumed, Busé's formula can be written

$$D = D_0 \log(aE + 1),$$

a and D_0 being constants depending on the properties of the plate and its development. A theoretical deduction of the formula by Silberstein is not very satisfactory, and the formula must be accepted as purely empirical. Nevertheless, its simplicity and its representation of certain observed features of the blackening curve make it of considerable use in practical work.

The action of individual photons on the individual grains has been investigated both from the theoretical and practical points of view, and it has been shown that the presence of silver sulphide in the silver bromide grains is essential if they are to be sensitive to light. Other impurities also appear to be essential. Eder and others have indicated that the ripening process consists in the reduction of the surface of the grain to silver, so that when the surface is dissolved away, the light sensitivity is strongly diminished (Luppo-Cramer), although the x-ray sensitivity is hardly affected. Sensitivity-nuclei specks have been predicted theoretically, and have been observed experimentally, although their role does not appear to be that initially ascribed to them. It is of interest to note that the sensitivity nuclei do not appear to be affected by x rays, any slight residual effect being most probably due to secondary effects.

In a recent paper Gurney and Mott (8) have suggested a mechanism for the formation of the latent image which is based on modern atomic physics, and which appears to be in good agreement with experimental results. The primary process of latent-image formation is the liberation of electrons within the grain due to the photo-conductive effect. These electrons move about in the lattice until they are trapped in the sensitivity specks. The field thus created will attract the silver ions, and the resultant speck of silver formed, if of sufficient size, will be stable and form the latent image. If the light intensity, however, is below a minimum value, the speck will be unstable, and the effect of the light will not be conserved.

Experiments have been conducted by Trillat and Motz (9) and Trillat and Merigoux (10) with a view to detecting the liberated silver. Using x rays they were only able to detect the presence of crystalline silver in strongly solarized plates (exposed to light for 10 days or more), the amount of free silver increasing with exposure time.

Examination of thin films of silver bromide by electron diffraction failed to show any signs of either crystalline silver bromide or silver even after exposure to light. This they ascribed to excessive bromide. When this was removed strong diffraction rings due to AgBr were found to occur, but no trace of silver. Using true emulsions containing gelatine, rings due to crystalline silver were found to appear after exposures of several seconds. If, however, silver bromide was detected in the beam, no silver appeared even after several minutes of exposure. It was therefore concluded that only in those AgBr crystals sufficiently distorted to prevent the formation of clear electron-diffraction patterns was any appreciable amount of crystalline silver liberated by the action of incident radiation. The sensitivity of a photographic grain to light thus depends essentially on its degree of perfection as modified by the presence of impurities and by the different manufacturing processes.

There appears to be an essential difference between the action of light photons and x-ray photons on photographic plates. In the former case the degree of perfection of the crystalline lattice is of primary importance, while in the latter any such effect is only slight. Moreover, several light photons are necessary to render a grain developable, while it is generally assumed that a single x-ray photon is capable of doing so. Owing to the different degrees of absorption in the grain, light will act mainly on its surface, while X rays will affect the whole of its volume. In view of these essential differences no exact analogy between the two types of reaction can be expected. Since we are essentially concerned with high-energy electronic radiation, the behaviour in X-ray work is of great interest, the action of light photons being more analogous to that of low-voltage electrons.

§ 3. X RAYS

The action of X rays on photographic plates was one of the first intimations of these rays, and its use in modern X-ray technique is widespread in scientific, medical and engineering work. Although the action of X-ray photons on photographic emulsions is probably not more complex than that of light photons, the experimental results obtained do not at first appear to be as consistent as for the latter.

An early investigation by Friedrich and Koch (11) served to trace the main outline of photographic action of X rays. The {blackening, density} and {blackening, log density} curves were found to be superficially similar to those obtained for light, but for the more energetic photons the blackening is proportional to the incident radiation, even at very low intensities. Moreover, the $\{D, \log E\}$ curve is much steeper than for light radiation, so that the linear portion of the plot occurs at much higher values of D. Most intensity measurements must therefore be carried out over the curved portion of the characteristic $\{D, \log E\}$ curve.

In 1920 Bloch and Renwick (12) published a useful summary of the applications of X-ray photography. In the course of the survey they pointed out the effect of development time on the plate, the photographic density at any point increasing with the period of development according to the formula $D = D_\infty(1 - e^{-kt})$. The formula appears to be true for a large range of developers as well as for different forms of incident radiation.

Glocker and Traub (13) extended the investigations of the $\{D, \log E\}$ relationship to the region of higher densities not already covered by Friedrich and Koch. Instead of the intensity of the incident beam being varied, with constant exposure time, they maintained the intensity constant, but varied the time of exposure. The relation thus established is between D and $\log t$, but in view of the validity of the reciprocity law (Glocker and Traub found $p = 0{\cdot}98 \pm 0{\cdot}01$), the more general relation between D and $\log E$ is immediately deducible. In the blackening curve thus found, three regions of blackening could be distinguished :

1. The region of under-exposure, $\Delta D \propto \Delta t$.
2. The region of normal exposure, $\Delta D \propto \Delta \log t$.
3. The region of over-exposure, where ΔD tends to zero or may become negative with increase in t (solarization).

Even at low intensities, however, blackening D and exposure E were found to be proportional, so that for X rays there is no initial lag in the response of the photographic plate.

A remarkable fact established by these experiments is that the blackening of a photographic plate by x rays is independent of the wave-length. A series of $\{D, \log E\}$ curves for varying wave-lengths is shown in figure 3, and they are seen to coincide within experimental error. Since the intensities of radiation of the different wave-lengths were not compared, however, the different curves might well be displaced relative to each other along the $\log E$ axis.

Bouwers (14) published a similar set of observations in 1923. At higher intensities he found that his results agreed closely with the empirical formula, $D=C\log(t/T+1)$, found by Busé to hold for light radiation. The curve thus obtained is very similar to those obtained by Blau and Altenburger and by Silberstein, satisfying the equation $D=D_\infty(1-e^{-bt})$. Bouwers also showed that for x rays the Schwarzschild constant $p=0{\cdot}99\pm0{\cdot}02$, so that the reciprocity law may be assumed to be valid. For varying wave-lengths C does not vary, but

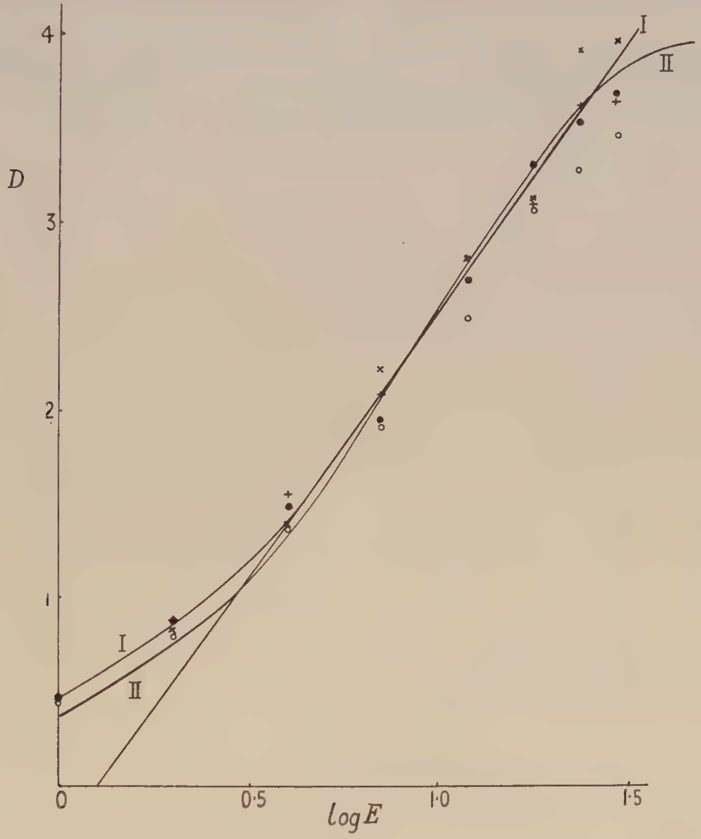


Figure 3. Characteristic curve for x rays.

× $\lambda=0{\cdot}87$ A. (Sr). ○ $\lambda=0{\cdot}56$ A. (Ag). + $\lambda=0{\cdot}47$ A. (Sb).

● $\lambda=0{\cdot}39$ A. (Ba) (Glocker and Traub).

I. Theoretical curve assuming full absorption.

II. „ „ „ negligible absorption.

the exposure times must be in a suitable ratio if the curves are to coincide. At low intensities Bouwers did not find the photographic density proportional to the exposure. In a subsequent investigation, however, Brindley and Spiers (15) found that D and E were proportional for values of D as high as 1·4. Berthold and Glocker (16) have also investigated the photographic action produced by the same quantity of radiation at various wave-lengths and found sharp discontinuities at the absorption edges of silver and bromine. Vogel and Clark estimated that 300 silver atoms are necessary to render a grain developable. In a collision an α particle liberates about 50 000, an x-ray photon about 1000, while a light photon can only liberate a single atom. The action of x-ray photons on photographic plates is thus simpler than that of light photons, since one x-ray photon is generally sufficient to render a grain developable. Silberstein and Trivelli (17) therefore suggested that the formula

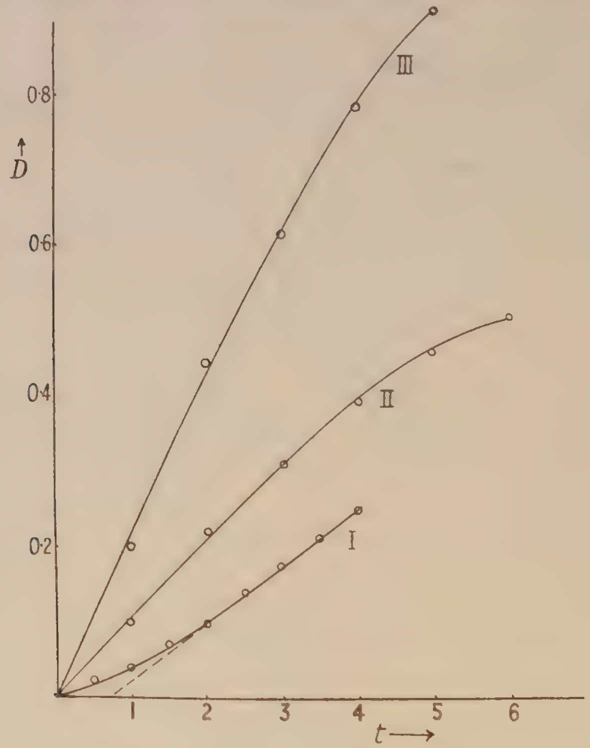


Figure 4. Characteristic curves for very soft x rays (Broili and Kiessig) showing lack of initial linearity for fine-grained plates. λ=45 A.

$$K=N(1-e^{-na\varepsilon}),$$

which had been found unsatisfactory for light rays, should apply to x rays. The formula was tested for a series of grain sizes, thin (single layer) plates being used, and agreement was found to be satisfactory.

In the course of a review of the measurement of x-ray spectra, Blake (18) gives an interesting summary of the preceding work, and its application to the measurement of X-ray intensities. Many of the methods adopted by previous workers in the field are included and their conclusions discussed.

Bell⁽¹⁹⁾ has recently published the results of a comprehensive investigation into the photographic action of X rays, with special reference to methods for measuring the film speed. He found that Busé's equation represented the blackening curves with a fair degree of accuracy and that the formula suggested by Blau and Altenburger,

$$D = A(1 - e^{-bt}),$$

although somewhat less satisfactory, could also be used. As had been found by previous investigators, the "toe" of the $\{D, \log E\}$ curve is much longer for X rays; thus, for an ordinary plate, Bell found that whereas the linear portion of the $\{D, \log E\}$ curve occurs for values of the blackening between 0·75 and 1·8 in the case of light rays, it extends between 2 and 3·5 in the case of X rays. Moreover, with double-coated X-ray emulsion plates, the linear portion only commences at $D = 3$. The shape of the $\{D, \log E\}$ curve was found to be independent of the wave-length of the incident radiation, although it was necessary to multiply the

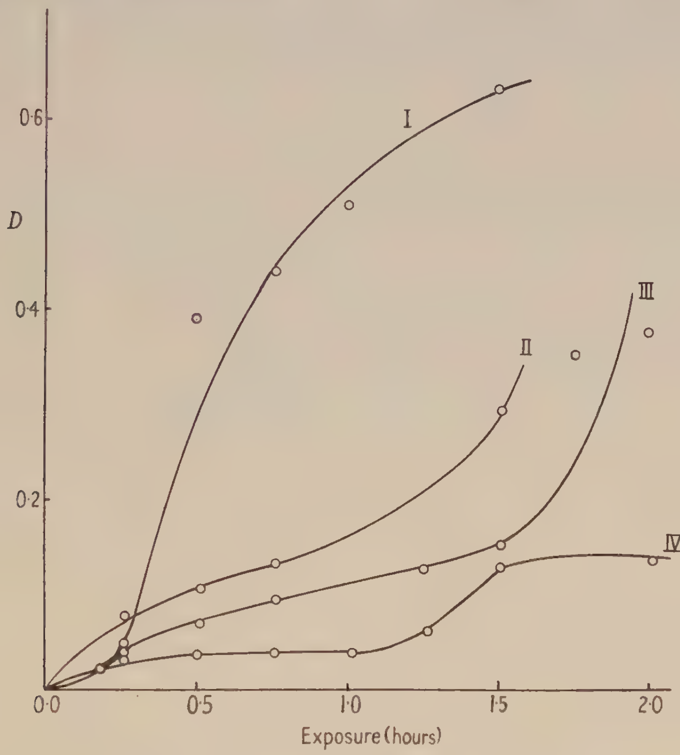


Figure 5. Blackening by very soft X rays (Hirsch).
I. Ga $L\beta_1$; $\lambda = 11{\cdot}006$ A. II. Zn $L\beta_1$; $\lambda = 11{\cdot}958$ A.
III. Cu $L\beta_1$; $\lambda = 13{\cdot}029$ A. IV. Ni $L\beta_1$; $\lambda = 14{\cdot}23$ A.

intensities by constants depending on the wave-length to make the different curves coincide. Bell also showed that the reciprocity law is valid for a hundred-fold variation in t, but showed slight deviation over a range of one thousand.

In view of the difference between the type of the photographic action in the case of X-ray photons and light photons, it has become of interest to investigate the intermediate region, where the change from the multiple-particle to the single-particle type of reaction occurs. Broili and Kiessig (20) were able to show that for 45 A. (270 v.) X rays, blackening and radiation density were no longer proportional at low values of the latter, at any rate for fine-grain plates (figure 4). Hirsch, moreover, has been able to obtain a series of blackening curves in the region 11 to 14 A. which are unlike those due to the more familiar types of radiation (figure 5). His results will be discussed in the following section in connection with a similar set of curves obtained with electron radiation.

§ 4. ELECTRONS

The action of electrons on photographic plates is of importance not only in electron-diffraction work, but also in other branches of electronics, e.g., oscillography. In most cases interest is mainly devoted to electrons of energies between 10 and 100 kv., and for these the results obtained appear to be most firmly established. Moreover, the high energy of the particles ensures that their action on the photographic emulsion is similar in many respects to that of X-ray photons, one particle being assumed sufficient to render a grain developable.

Becker and Kipphan (22) have published the results of an investigation with electrons of between 18 and 84 kv. After suitable corrections for fluorescence evoked by the passage of the electrons, the reciprocity law was found to be valid to a high degree of accuracy, a result previously obtained by Bothe for β rays and by Nacken (23) for 1500-v. electrons. Of the three types of plate used, two (Agfa Extra-Rapid and Matter plates) were somewhat similar in their properties, while the emulsion of the third (Q plates) was considerably thinner and contained very little gelatine. Some of the blackening curves obtained (figure 6) appeared to satisfy Busé's equation $D = C \log(aQ+1)$, and showed an extensive linear relation between D and $\log E$. At low densities D and E are proportional, a result in agreement with that obtained for other forms of high-energy radiation (X rays and α particles). With Q plates the linear relation between D and E extends to values of D as high as 1·5. The effect of increasing the voltage of the beam is to increase the blackening, although with Agfa Rapid plates a saturation effect becomes noticeable at about 70 kv. This is ascribed to the increasing penetration of the electron beam through the emulsion, since the thicker Matter emulsion showed no such effect.*

The sensitivity of a plate may be defined by the ratio D/E of the blackening to the incident radiation, and will thus be constant for small values of the blackening,

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* At 55 kv. it has been estimated that about 20 per cent of the incident radiation traverses the emulsion, the proportion increasing with increasing voltage.

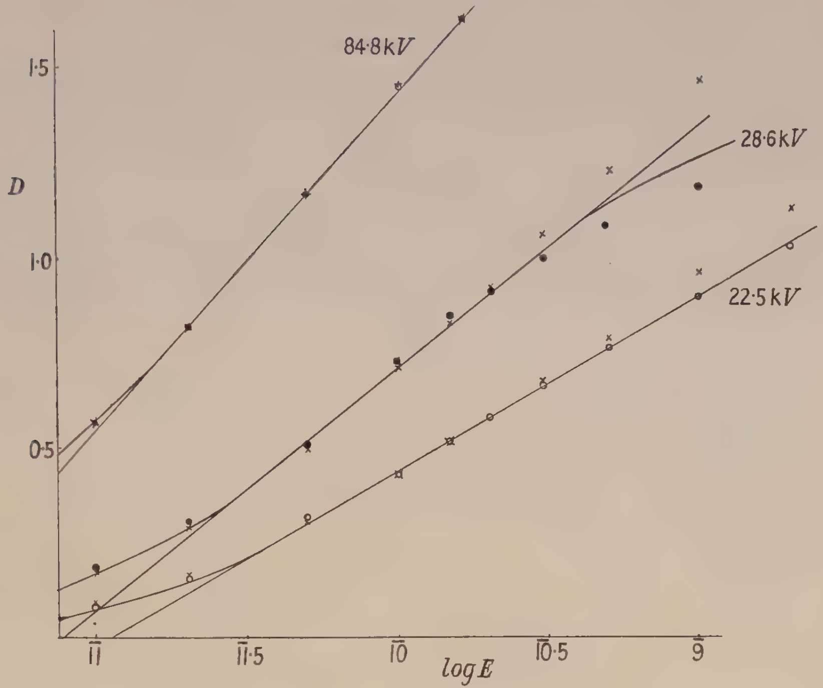

Figure 6. Blackening by medium-voltage electrons.

○ ● Experimental values (Becker and Kipphan).

× Values from Busé's formula.

—< Theoretical formula and asymptote.

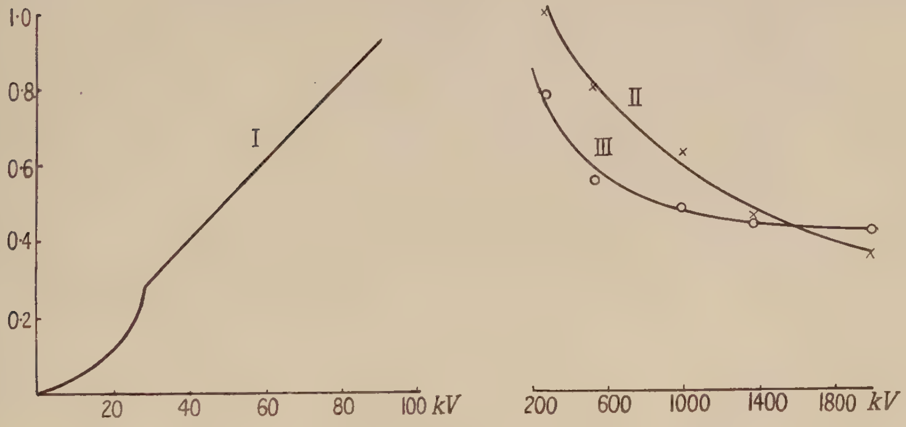

Figure 7. Variation of sensitivity with electron voltage.

I. $10^{11} \times$ blackening/charge for small densities (Becker and Kipphan).
II. Relative photographic action (Ellis and Aston).
III. Relative ionization.

subsequently decreasing as *D* and *E* increase. Figure 7 shows that over the range 25 to 75 kv. the sensitivity increases uniformly, with a sudden change in the neighbourhood of 25 kv. (probably due to X-ray absorption by the silver atoms) and a maximum at about 100 kv., the sensitivity subsequently decreasing.

Similar results have been obtained by Schäffer (24) and by Borries and Knoll (25), although the former finds that over the range 30 to 65 kv. the variation of sensitivity with voltage is smaller than that obtained by Becker and Kipphan.

Weidner (26), using similar plates, investigated the region of electron energies below 800 volts. With gelatine plates, sensitivity levels were found to occur at 35 and 60 volts, while in no case were electrons of less than 22 volts energy found to have any photographic effect. Between 50 and 60 volts there is a very rapid variation of blackening with increasing voltage, while above 60 volts the variation becomes slight.

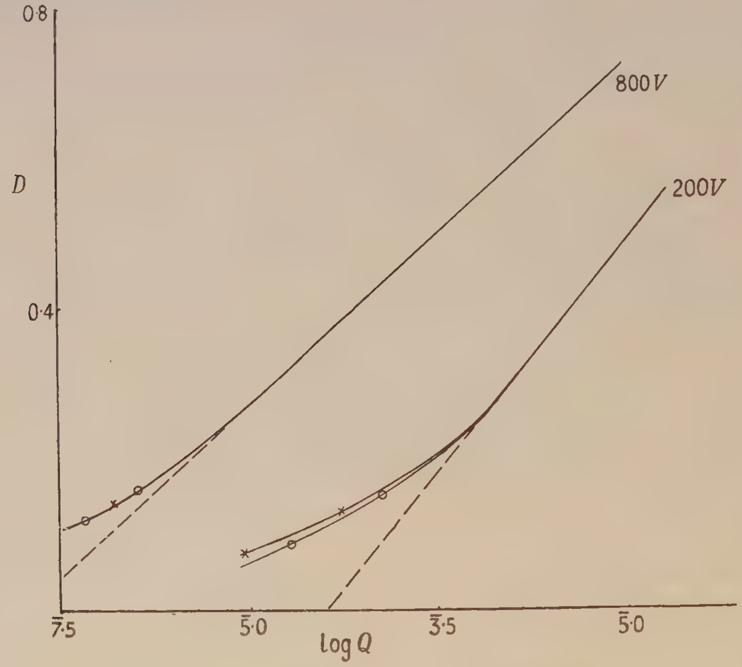

Figure 8. Blackening with low voltage electrons.

○ Experimental values (Weidner).
× Theoretical curve (full absorption).
Experimental and theoretical curves adjusted along the same linear portion.

The reciprocity law was found to be valid, although some exceptions were noted. The blackening curves obtained are shown in figure 8 and are similar to those obtained at higher voltages. The initial sensitivities are compared in Table 1 with those obtained by Becker and Kipphan at higher voltages. It appears that as a very rough approximation the blackening depends only on the total energy (*VE*) of the incident beam.

Light photons are able to affect photographic plates at energies well below 22 electron volts. Burroughs (27) was only able to obtain blackening with 21-volt

Table 1. Initial sensitivity Q/D when D is small

Volts	Matter plate coul./cm^2	Agfa plate coul./cm^2	Q plate coul./cm^2
40	$28{\cdot}8\times10^{-6}$	$36{\cdot}0\times10^{-6}$	$58{\cdot}6\times10^{-6}$
80	13·0 ,,	17·6 ,,	15·6 ,,
200	13·0 ,,	17·6 ,,	5·2 ,,
400	8·1 ,,	12·14 ,,	2·26 ,,
800	2·3 ,,	6·06 ,,	1·26 ,,
1130		3·38 ,,	0·78 ,,
kv.			
18	$1{\cdot}65\times10^{-10}$	$1{\cdot}65\times10^{-10}$	$13{\cdot}66\times10^{-10}$
22·5	1·11 ,,		
28·6	0·40 ,,		3·07 ,,
30·2		0·38 ,,	
53·7	0·18 ,,		1·33 ,,
64		0·17 ,,	
93	0·11 ,,		
98		0·12 ,,	

electrons in the presence of mercury vapour, owing to its fluorescence. Meier (28), however, using Schumann plates, claims that at much higher electron intensities blackening is obtainable at voltages as low as 5 volts.

The high-energy electrons may well be compared with x-ray photons, and the low-energy electrons with light photons, since each of the former has sufficient energy to render a photographic grain developable, while several of the latter are necessary to do so. The intervening region is of interest as it indicates the form of the transition. This region has been studied for electrons by Seitz and Harig (29) and by Nacken (23). Since the only obvious difference between the two types of reaction is the lack of initial proportionality between D and E in the less energetic particles and the steeper slope of the $\{D, \log E\}$ curve with the more energetic ones, one might perhaps expect the transition to be gradual, the initial minimum exposure E gradually diminishing while the slope increased with increasing particle energy. Figure 9 shows, however, that, for electrons at any rate, this is by no means the case. Starting at low energies the $\{D, \log E\}$ curve shows the presence of a minimum effective exposure. As the electron energy increases, an inflexion appears which gradually increases in size, finally extending over the major portion of the curve. It thus appears that the linear regions of the $\{D, \log E\}$ curves for low- and for high-energy electrons are essentially different in origin and nature, corresponding to the two different types of reaction. It has been independently estimated that 300 silver atoms are necessary to render a grain developable, and if each of these requires 25 electron volts the transition between the two types of reaction might be expected to occur at 7500 volts. As is seen in figure 9, this is a good estimate of the transition energy. Seitz and Harig explain the two portions of the curve as being due, one to the superficial blackening of the emulsion by low-range electrons and the other to the volume blackening by the longer range secondary x-rays generated. The photographs they publish of the cross-section of a developed emulsion do not seem to support this view. Moreover, similar types of blackening curves have been obtained by Hirsch (figure 5) with photon radiation intermediate between x rays and light, which thus plays an analogous part to the medium-energy electrons used by Seitz and Harig and for which no such explanation can be advanced.

It therefore appears that the transition from the multiple-particle to the single-particle type of photographic action is accompanied by a complete distortion of the blackening curve instead of by the gradual transition which might be expected.

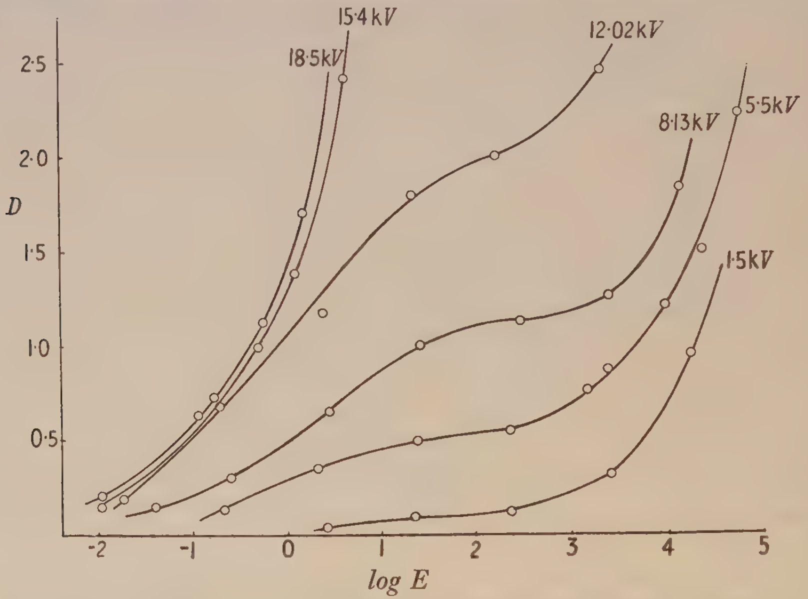

Figure 9. Blackening by electrons at intermediate voltages (Seitz and Harig).

Any comprehensive theory of photographic blackening should therefore be capable of explaining the transition as the number of particles necessary to render a grain developable is decreased.

Investigations into the photographic action of electrons of very high energies have been carried out by Ellis, Aston, Wooster, Bethe, Salbach, Svedberg Anderson and others, for both homogeneous and heterogeneous radiation. Thus Svedberg and Anderson (30), using thin layer plates, found the percentage of developable grains of various sizes after a series of exposures to β radiation, the corresponding blackening being in approximate agreement with the simple formula $D = D_\infty(1 - e^{-na\varepsilon})$.

Bethe (31) found that the reciprocity law was obeyed to a high degree of accuracy since p was $1{\cdot}00 \pm 0{\cdot}01$. The blackening curves obtained were similar in many respects to those for x rays, and in particular there was a linear relation between D and E for blackening up to D equal to 0·9. Salbach (32), on the other hand, found that the value to be ascribed to $q(=1/p)$ was 0·89, but the results were not very consistent.

Ellis and Wooster (33), using homogeneous radiation, found that the validity of the reciprocity law could be assumed. The blackening curves obtained could be represented by the Busé equation

$$D = C\log(1 + It/T),$$

where C depended on the development time and T on the energy of the incident electrons, increasing with the latter. Thus at constant development time the effect of an increase in the electron energy could be compensated by a suitable increase in the exposure time, a result similar to that which holds for blackening by x rays. It was also suggested that the photographic activity of the β rays is directly proportional to the energy lost in the film, i.e. to $1/\beta^2$. In a subsequent paper, however, Ellis and Aston (34) showed that above 200 kv. the photographic action decreases more rapidly than $1/\beta^2$, but at lower energies the decrease is less rapid. Since in the region of 10 kv. to 80 kv. the photographic action increases with increasing energy, a region of maximum sensitivity might be expected to occur in the neighbourhood of 100 kv., the actual value depending on the particular type of plate concerned. This maximum sensitivity occurs at about 70 kv. with Agfa Special Rapid plates, but at a higher value with Matter plates. An approximate representation of this variation is shown in figure 7.

Similar results have been obtained for positrons by Bewilogua and Dixit (35) and by Thibaud (36). The former deal with the absorption of high-velocity positrons, but no blackening curves are given. Thibaud was able to show that the absorption is exponential in nature, a result of interest to the subsequent theory.

A number of experiments have been carried out on the blackening of photographic plates by electrons, with some more particular aim in view. Thus in electron diffraction, blackening curves have been obtained by Thomson (37); Mark and Wierl (38); White (39); Shirai (40); Yearian (41); and Ornstein, Brinkman, Hauer and Tol (42). Marton (44) compared the sensitivity of a series of different plates at voltages between 50 and 100 kv. with a view to estimating their suitability for electron microscopy. Relative to Gevaert Process Plates as a standard, it was found that x-ray films were more sensitive at low intensities and Process plates at higher intensities.

§ 5. EXPERIMENTAL DETERMINATION OF THE BLACKENING CURVE FOR ELECTRONS

The relation between photographic blackening and electron density must in general be determined in any measurement of intensity involving the photographic plate. Attempts (45) have been made to avoid this difficulty, e.g. by exposing a series of plates for different time intervals (t_1, t_2) so chosen that the densities (D_1, D_2) of the points whose electron intensities (i_1, i_2) are to be compared are equal. Then, assuming the validity of the reciprocal law, i.e. that to equal densities (D_1, D_2) correspond equal electron densities ($E_1 = E_2$), it follows that the electron intensities (i_1, i_2) will be inversely as the times necessary to reach the standard photographic density ($D_1 = D_2$). The method is obviously only of limited use and cannot readily be extended. In the ideal case the relation which exists between photographic blackening and electron density (calibration curve) should be established by exposing a series of identical photographic plates for the same time t to an electron beam of constant voltage V, the intensity being varied for each plate. The development of these plates must then proceed in some standard fashion. The {blackening, electron density} curve thus found can only be applied to a similar plate exposed to the same voltage beam for the same period of time, t, but the results are independent of the validity of the reciprocal law. The law has been shown by several investigators to be valid over a wide range of exposure times and may well be assumed to hold generally for electrons.

Unfortunately, the method described suffers from a number of serious objections. The restriction on the voltage and duration of the incident beam is not of great importance, as electron-diffraction patterns are usually taken at approximately the same voltage, while the time factor may be allowed for if the validity of the reciprocal law is assumed. On the other hand, many electron-diffraction cameras do not allow the intensity of the beam to be varied without a corresponding voltage variation, and an accurate measurement of the incident beam intensity is not always easy, owing to its small value and to the presence of spurious charges from the high-tension plant. The various plates will, moreover, vary to an appreciable extent in their properties, while the necessity of identical development for all plates is attended by some experimental difficulty. Finally, only a limited number of points on the calibration curve are obtainable.

The methods adopted in the course of the author's experiments described below are based on the validity of the reciprocity law over a limited range. The same pattern was taken on a plate, the two halves of which were exposed for two different and known time-intervals t_1 and t_2. This could easily be done with the split-shutter plate holder, a standard accessory in the laboratory. The times t_1 and t_2 measured with a stop-watch varied between about 10 and 50 seconds. Patterns of the type shown in figure 10 were thus obtained.

Let E_A denote the electron density (number of electrons per sq. cm.) at the point A. In what follows we shall assume that the electrons forming the background (diffuse scattering) have the same voltage as the main diffracted beams, as has been shown to be the case by Nie (46). It is then possible to use this gradually varying intensity for the calibration curve. In the first method adopted let A, a refer to points near each other, but such that A is in the half-plate exposed for time t_1, while a is in the half-plate exposed for time t_2. A and a may best be chosen near the boundary of the two halves, equidistant from it and from the central point. Let B, b; C, c; etc., be related in the same way.

Since A, a are equidistant from the central spot and are very close, the ratio of electron densities $E_A/E_a=t_1/t_2$. It is assumed that neither A nor a lies very close to a diffracted spot. To these electron densities correspond the microphotometer deflections d_A and d_a. The plate is now moved in the photometer until a second point B is found such that $d_B=d_a$, and thus $E_B=E_a$ within the limits

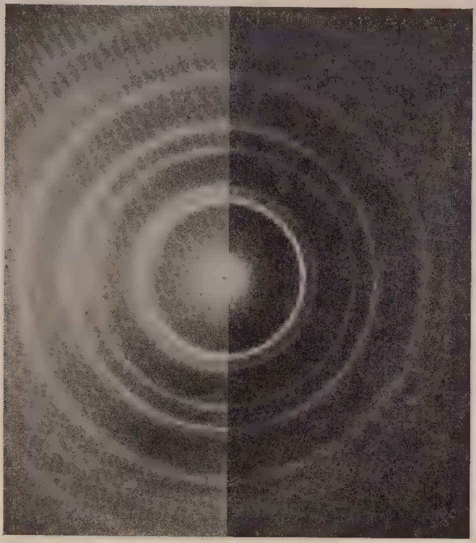

Figure 10 *a*. Type of electron-diffraction pattern used for calibration.

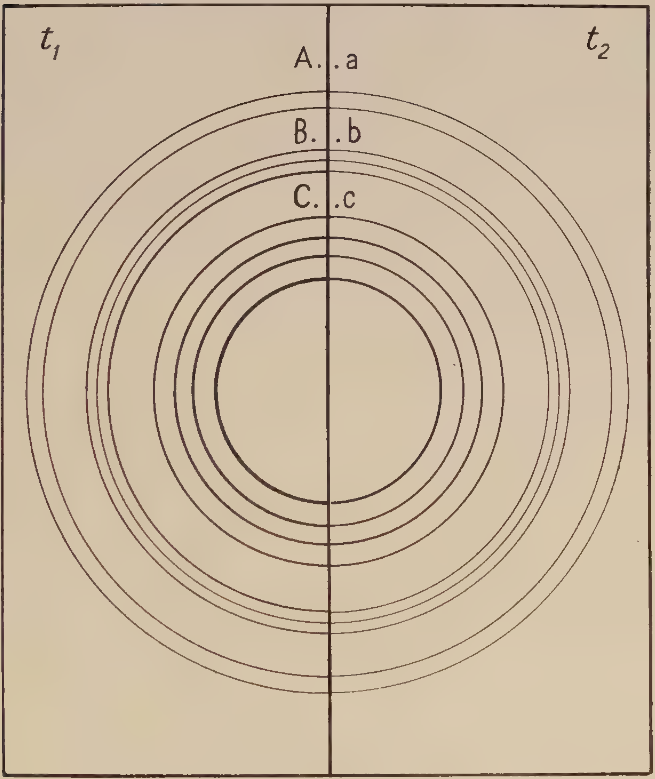

Figure 10 *b*. Double exposure pattern.

permitted by the reciprocity law. At the point b near B a deflection d_b will be found, corresponding to an electron density E_b. We now choose a point C such that $d_C = d_b$ and continue the process. In this way a range of microphotometer readings d_A, $d_a = d_B$, $d_b = d_C$ are found corresponding to the electron densities E_A, $E_a = E_B$, $E_b = E_C$, these being in the known ratio t_1/t_2. A calibration curve of d against E may thus be plotted. If a different point A' is chosen initially and another calibration curve drawn up, both calibration curves should coincide by suitable translation parallel to the E axis.

The second method adopted is to take a microphotometer record along a line passing through the undeflected beam spot, so that the two halves of the curve correspond to exposures t_1, t_2 and corresponding points on them will have electron densities in the ratio t_1/t_2. The procedure described above is then adopted in the graph itself, either to the smoothed curves or to the background, which may be interpolated where necessary.

The first method (Edge-Comparison Method) is very rapid and depends solely on the hypothesis that the background density at a given radial distance does not vary rapidly along the circumference. Unfortunately, statistical grain size and frequency fluctuations at low intensities can only be averaged out if several calibration curves are made. Errors may occur when the distance Aa becomes comparable with the radial distance (i.e. near the origin). On the other hand, local variations in the properties of the emulsion and of its development will be less appreciable. Moreover, if the deflections d_A, d_B, d_C, etc. are in a diminishing sequence, initial errors will have little effect on subsequent measurements.

The second method (Continuous-Curve Comparison Method) is more tedious. Local grain variations are averaged out, but errors due to a slight error in the position of the central spot may assume some importance, as they are cumulative. Moreover, the assumption as regards the symmetry in the background distribution about the central spot is not necessarily justifiable.

The two methods outlined, and other obvious modifications, display several advantages over the use of a series of plates. Since the calibration is taken on the same plate as the pattern itself, variations between different plates are of no importance, and even the effect of variation of the emulsion properties over the plate itself is to some extent diminished. The otherwise stringent rules concerning development, etc., can be almost entirely relaxed, the only condition remaining being that no local variation in the strength of the developing solution should occur, and even this may be of minor importance. Since the two exposures are taken within a few seconds of each other, variations in the conditions of exposure are much less liable to occur. The only factors on which the accuracy of the measurements depend are essentially the time measurements t_1 and t_2, the steadiness of the incident beam, the uniformity of the photographic emulsion and the validity of the reciprocal law over the range considered. The first of these conditions is under the ready control of the experimenter, while the second can be checked by means of a Faraday cylinder. The plates used in the experiments were

Ilford Special Rapid plates developed in metol-hydroquinone developer for periods of about four minutes, this period not being critical for the methods adopted.*

In the plates taken in the case of the diffraction experiments, the most important measurements nearly always occurred near the centre of the plate, the regions of the plate near the edge being very lightly blackened. Errors due to edge effects, statistical fluctuations in blackening, etc., are appreciable for small densities only, where the calibration curve is generally of least interest.

The microphotometer used was of the hand-operated type. The photographic plate could be clamped in any position and moved both vertically and horizontally in directions normal to the scanning light direction. The light source consisted of a 12-volt 3-amp. bulb run off the mains by means of a suitable resistance with a steadying 12-volt accumulator in parallel. The light from this bulb was passed through several lenses and focused by means of a microscope collimator on to the part of the photographic plate under examination. The light transmitted through the plate was gathered by a microscope objective (16 mm. focal length, NA $=0{\cdot}28$) fixed to a vertical illuminator. Thence it was thrown on to the photocell through a diaphragm, the size of the hole determining the area of plate scanned. A part of the light passing through the vertical illuminator was tapped off by a glass slip and could be examined by means of a microscope eyepiece, enabling the illuminated part of the plate to be closely examined. The photocell used was of the secondary-emission vacuum type working up to 800 volts. The output of this, passed through a sensitive galvanometer, gave ample deflections (up to several hundred mm.), even with a very dark plate and a scanning hole smaller than the cross-section of the incident electron beam.

The linearity of response of the whole apparatus was tested by means of an optical wedge and was found to be satisfactory.

The plot of galvanometer deflection d against electron density E is not entirely satisfactory. Since (due to linearity of instrument response) blackening $D = \log d_0 - \log d$, where d_0 is the deflection corresponding to the "clear glass" portion of the plate, a plot of $\log d$ against E should give a straight line if $D = \alpha E$, but this is only true for low values of E. A far more satisfactory method of plotting was that of $\log d$ against $\log E$ (i.e. D against $\log E$), this method being generally used in similar characteristic curves for light and X rays. Since $\log d_0$ is a constant depending on the instrument, lamp brightness, etc., its measurement was often omitted. Over a remarkably long range the relation between $\log d$ and $\log E$ was found to be linear, and for the first batch of plates considered the slope was very nearly $-1{\cdot}33$. Later batches of plates showed, however, no such constancy from plate to plate.

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* The influence of development on the uniformity of a photographic plate may perhaps be emphasized. Dobson, Griffith and Harrison (47) found considerable density variations near the sides of uniformly exposed photographic plates (edge effects). Even with a developing tank containing a false bottom as recommended by Ilford, Ltd., a density variation of from 0·53 to 0·59 was noted, i.e. an error of ±5 per cent. With a special type of tank they were able to reduce this variation to one of between 0·55 and 0·57 (2 per cent). Bouwers (14) similarly claims results agreeing to within 3 per cent of mean if readings are taken more than 2 cm. from the edge. This restriction has perhaps not always been observed by other experimenters.

Several examples of such plots are shown in figure 11. In a number of cases the calibration curves found by the use of both methods described above were compared, and no systematic errors detected. The curves were also compared with the theoretical relation calculated below, and the agreement between them is satisfactory.

An extension of these methods to the photometry of positive plates may lead to a further simplification of the processes necessary for the measurement of electron-diffraction intensities. These plates are obtained by exposing a photographic plate in contact with the negative of the pattern to a light source for a suitable time. These positive plates are then photometered. Since they consist of two half-plates of known relative intensities, a calibration curve can be readily established. A great advantage of this method of procedure is that to increasing electron densities in the pattern correspond increasing deflections,

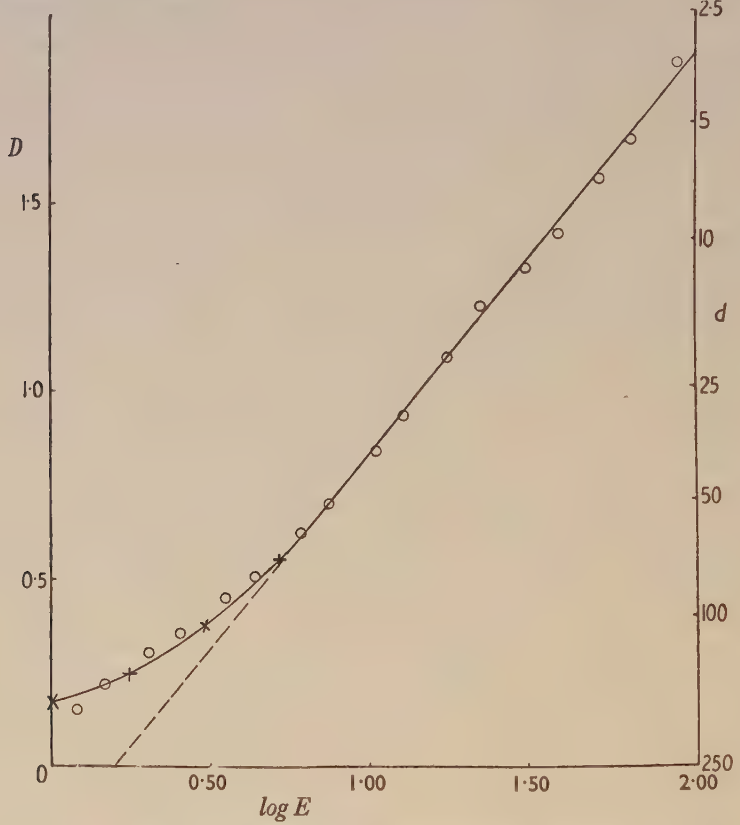

Figure 11 *a*. Blackening curve for 50-kv. electrons.

○ Experimental curves. × Theoretical curve.
d Galvanometer deflection. *D* and *E* have arbitrary origin.

so that the sensitivity of the instrument remains appreciable even for high intensities.

As is well known, over a wide range there exists a linear relation between the density D_p and the logarithm of the light exposure I for plates exposed to light. Thus we may write

$$D_p = \log \delta_0' - \log \delta' = \gamma \log I + C,$$

γ and C being constants for a given development time and plate. Moreover, a similar relation holds for the negative plates exposed to the electron beam E:—

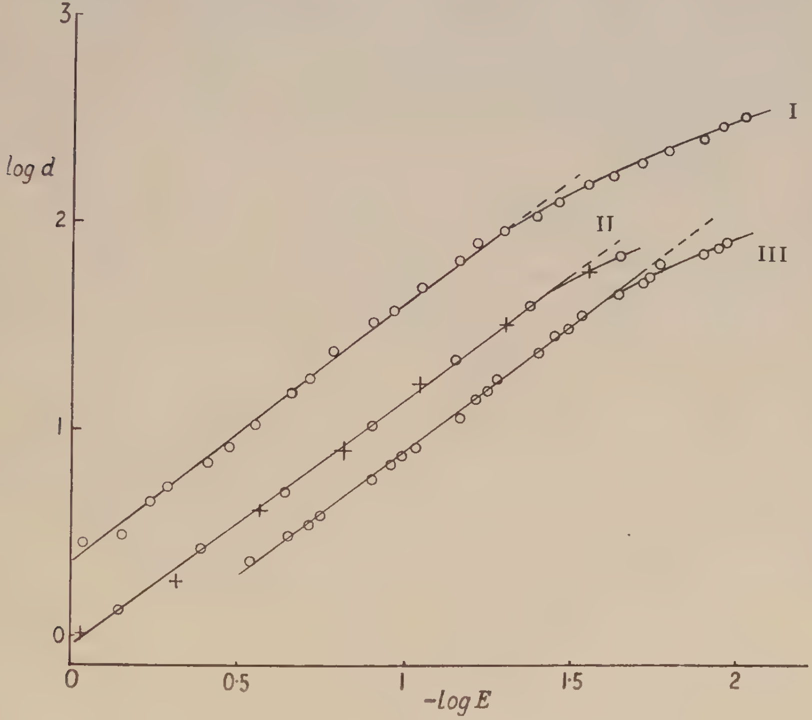

Figure 11 *b*. Blackening curve for 50-kv. electrons.
I: Edge-comparison method (3 sets of observations); II and III: Continuous-curve method (6 sets of observations).

$$D_n = \log \delta_0 - \log \delta = \alpha \log E + K.$$

In both these cases the letters δ refer to microphotometer deflections, which are assumed proportional to the intensity of the light-scanning spot in the photometer.

Owing to the method of preparation of the positive plates, it follows that $I = \beta\delta$, β being a constant. Therefore on replacing in the previous formulae and eliminating,

$$\log \delta' = \alpha\gamma \log E + \log A \text{ or } \delta' = AE^{\alpha\gamma}.$$

Thus if by suitable exposures and development times it is possible to obtain

$\gamma = 1/\alpha$, the light transmission of the positive plates (and consequently the galvanometer deflection) will be a direct measure of the electron density in the region

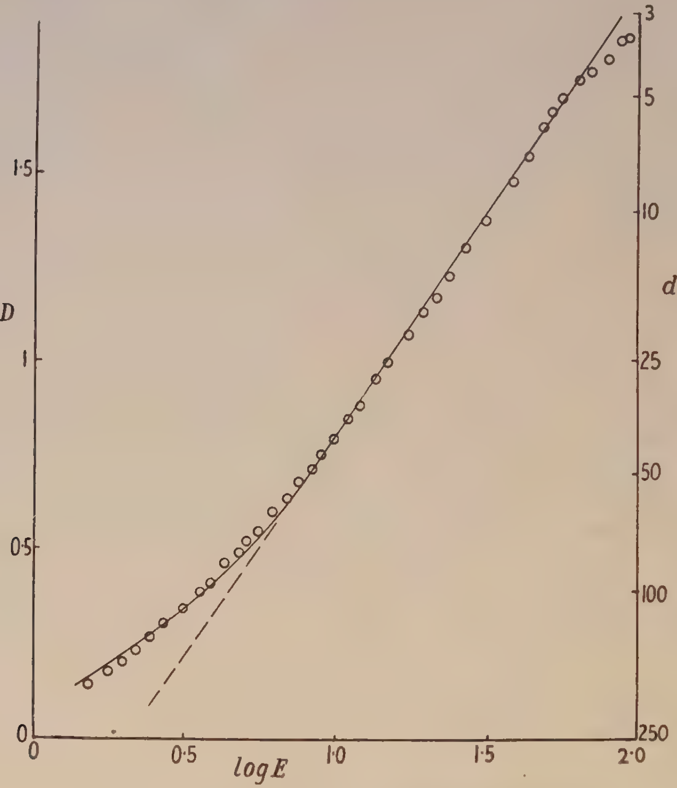

Figure 11 *c*. Blackening curve for 50 kv.-electrons.

o Experimental values.

——— Theoretical curve, assuming full absorption.

where the two linear positions of the $\{D, \log E\}$ and $\{D, \log I\}$ curves overlap. Even if this relation is not fulfilled, however, the measurement of electron densities is rendered both simpler and more accurate by the use of positive plates.

§ 6. THE ACTION OF ELECTRONS ON PHOTOGRAPHIC PLATES

In most theories of the photographic action of photons or electrons, the effect of absorption on the intensity of the beam has not been seriously considered. Nevertheless, this effect may well be of great importance, e.g. in the action of 50 kv. electrons on photographic emulsions of about 20 μ thickness. In the following theory an attempt has been made to take this absorption into account for the simplest possible cases. The formula found also appears to be applicable to other forms of radiation than electrons, and a discussion of its application is appended.

The effect of absorption on the electron beam is to decrease the number of impinging electrons as the depth within the emulsion increases, while the velocity and therefore ionizing power may also vary. For simplicity, we shall assume that when a collision with a photographic grain occurs, most of the electron energy is lost. Thus the electron density and ionizing power of the beam may be assumed to fall off exponentially. Any other hypothesis which leads to a similar variation of ionizing power with depth will lead to identical results.

The hypothesis introduced is not entirely unfounded, since Lenard has shown that the electron density falls off exponentially with thickness. Moreover, most of the photographic action of the electrons will occur at energies above the x-ray absorption level of the silver atoms at about 20 kv. Thus very few collisions per 50 kv. electron are possible. Over the range 20 to 80 kv. there is little difference in the photographic action per electron, so that the variation with velocity is small.

Consider an emulsion of thickness d, containing n_0 grains per unit volume, each of cross-sectional area a. Of these n_0 grains, let n become developable owing to the incident radiation consisting of E photons per unit area of the emulsion. At a depth x from the surface, the radiation will be equivalent to $Ee^{-x/r}$, r being the range of the electrons in the emulsion. For a layer of unit area and at the depth x,

$$dn=(n_0-n)adE_x=(n_0-n)adEe^{-x/r}. \qquad \ldots\ldots(1)$$

If the probability of a grain becoming developable when transpierced by an electron is, however, only ϵ, equation (1) becomes

$$dn=(n_0-n)a\epsilon dE_x$$

or

$$n=n_0[1-\exp(-a\epsilon E_x)]=n_0[1-\exp(-a\epsilon Ee^{-x/r})]. \qquad \ldots\ldots(2)$$

If the layer is of thickness Δx and of unit area the corresponding number of developable grains is given by $n_0\Delta x$.

The transmission of light through the layer may now be calculated, since it is this which determines the density of the plate in the microphotometer. Let $I(x)$ represent the light incident at depth x on unit area of the emulsion in the microphotometer. Then the amounts of light emerging through the emulsion not containing any grains, that containing undeveloped grains and that containing developed grains are, respectively,

$$I_f(x+\Delta x)=I(x)f(1-n_0a\Delta x),$$
$$I_g(x+\Delta x)=I(x)g(n_0-n)a\Delta x,$$
$$I_h(x+\Delta x)=I(x)hna\Delta x,$$

f, g, h being the transmission coefficients for these three types of medium Thus

$$I(x+\Delta x)=I(x)[f+(g-f)n_0a\Delta x+(h-g)na\Delta x].$$

With a clear plate $n=0$ and

$$I_0(x+\Delta x)=I_0(x)[f+(g-f)n_0a\Delta x].$$

Thus the variation in transmission due to $n\Delta x$ developable grains is given by

$$I(x+\Delta x)=I_0(x+\Delta x)\left[1-\frac{(g-h)na\Delta x}{f+(g-f)n_0a\Delta x}\right]$$

$$=I_0(x+\Delta x)\left[1-\frac{(g-h)na\Delta x}{f}\right] \qquad \ldots\ldots(3)$$

for small values of Δx. The light transmission coefficient is thus altered by a factor $1-An\Delta x/n_0$, where $A=\dfrac{g-h}{f}an_0$.

Thus $$I_E(x+\Delta x)=I_0(x+\Delta x)\left[1-\frac{An\Delta x}{n_0}\right]$$

$$=I_0(x+\Delta x)[1-A\Delta x\{1-\exp(-a\epsilon Ee^{-x/r})\}]. \qquad \ldots\ldots(4)$$

The total reduction in intensity

$$I_E(d)=I_0(d)\prod_{x=0}^{x=d-\Delta x}[1-A\Delta x\{1-\exp(-a\epsilon Ee^{-x/r})\}]. \qquad \ldots\ldots(5)$$

For a different value of the incident intensity $E'=Ee^{-\Delta x/r}$

$$I_{E'}(d)=I_0(d)\prod_{x=0}^{x=d-\Delta x}\left[1-A\Delta x\left\{1-\exp\left(-a\epsilon Ee^{-\frac{x+\Delta x}{r}}\right)\right\}\right]$$

$$=I_E(d)\frac{1-A\Delta x\{1-\exp(-a\epsilon Ee^{-d/r})\}}{1-A\Delta x\{1-\exp(-a\epsilon E)\}}. \qquad \ldots\ldots(6)$$

As $\Delta x \to 0$

$$I_{E'}(d)=I_E(d)\{1+[\exp(-a\epsilon Ee^{-d/r})-\exp(-a\epsilon E)]A\Delta x\}. \qquad \ldots\ldots(7)$$

The slope of the $\{\log I_E(d), \log E\}$ curve is thus

$$M[\exp(-a\epsilon Ee^{-d/r})-\exp(-a\epsilon E)]A\Delta x/(-M\Delta x/r)$$

$$=-Ar[\exp(-a\epsilon Ee^{-d/r})-\exp(-a\epsilon E)]. \qquad \ldots\ldots(8)$$

It is customary to use the $\{D, \log E\}$ curve where

$$D=\log I_0-\log I_E, \qquad \ldots\ldots(9)$$

so that the slope of the $\{D, \log E\}$ curve is given by

$$dD/d\log E=Ar[\exp(-a\epsilon Ee^{-d/r})-\exp(-a\epsilon E)]. \qquad \ldots\ldots(10)$$

In this formula $e^{-d/r}$ is the probability of an electron passing through the emulsion, and may be written α. Thus (10) becomes

$$dD/d\log E=Ar[\exp(-\alpha a\epsilon E)-\exp(-a\epsilon E)]. \qquad \ldots\ldots(11)$$

§7. PLOTTING OF THE CURVE

The form of the curve deduced will depend essentially on the value of the transmission coefficient α. Equation (11) cannot be integrated directly, but may readily be plotted by means of the differential equation

$$\frac{dy}{dx}=1-e^{-e^x}=1-e^{-z}, \quad \text{where} \quad e^x=z. \qquad \ldots\ldots(12)$$

The latter may be integrated and leads to the rapidly converging series

$$y=\int_{x=-\infty}^{x=x}\left\{1-\left(1-e^{x}+\frac{e^{2x}}{2!}-\ldots\right)\right\}dx=e^{x}-\frac{1}{2.2!}e^{2x}+\frac{1}{3.3!}e^{3x}-\ldots \qquad \ldots\ldots(13)$$

When x is large, the curve approximates to a straight line of unit slope. For smaller values of x it is strongly curved, having the x-axis as asymptote. The function is shown in figure 12. Since it is more convenient to plot logarithms

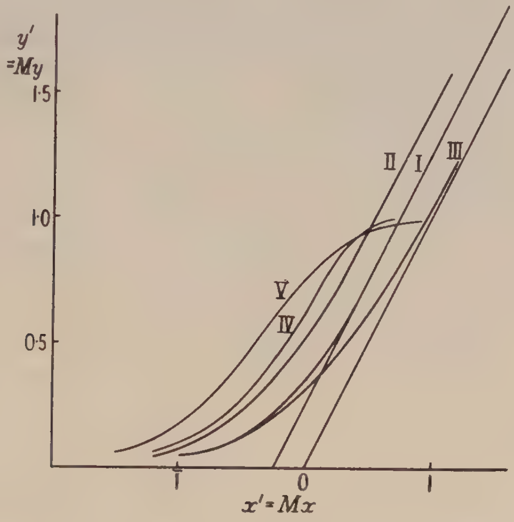

Figure 12. Theoretical curves ($y'=My$, $x'=Mx=\log z$).

I. $dy/dx=1-e^{-z}$ (equation (12)).

II. $y=\log_e(1+z)+1-\dfrac{1}{1+z}$ (equation (22)).

III. $y'=\log(1+z)$ (Busé formula).

IV. $y'=1-e^{-z}$.

V. $y'=1-1/(1+z)^2$ (equation (23)).

I is the theoretical formula, II the corresponding formula for an exponential grain-size distribution, IV and V are the corresponding formulae for negligible absorption.

to the base 10, one may put $x'=Mx$ and

$$dy/dx'=(1-e^{-10^{x'}})/M, \qquad \ldots\ldots(12')$$

the maximum slope now becoming $1/M$.

When $\alpha=0$ (full absorption), the relation between D and $\log E$ can be immediately deduced from (12′) by multiplying the ordinates of the latter by the amount MAr. Thus

$$y=D/MAr, \qquad x'=\log a\epsilon E, \qquad \ldots\ldots(14)$$

so that $D=MAry$ and $\log E=x'-\log a\epsilon$.

If, however, α is not zero, the ordinate y corresponding to a value x' may be obtained from the difference of the two ordinates y_2, y_1 corresponding to the abscissae x' and $(x'+\log\alpha)$, differing by the constant amount $\log\alpha$. Thus

$$\frac{d(y_2-y_1)}{dx}=(e^{-10^{x'+\log\alpha}}-e^{-10^{x'}})/M, \qquad \ldots\ldots(15)$$

where $y_2-y_1=D/MAr$ and $x'=\log a\epsilon E$, so that $D=MAr(y_2-y_1)$. Since the

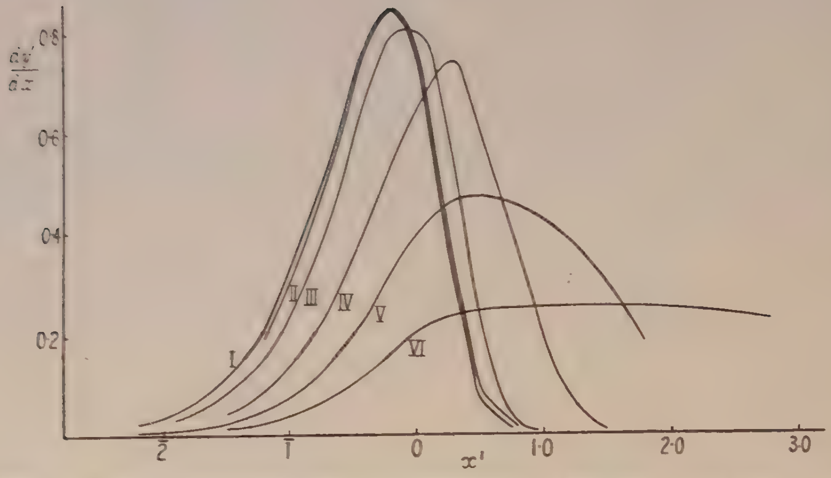

Figure 13. Slope of blackening curve for various values of the transmission coefficient α.
$dy'/dx'=(e^{-\alpha z}-e^{-z})/\log \alpha$,
$\alpha=1{\cdot}0$ (I), $0{\cdot}9$ (II), $0{\cdot}5$ (III), $0{\cdot}1$ (IV), 10^{-2} (V), 10^{-4} (VI).

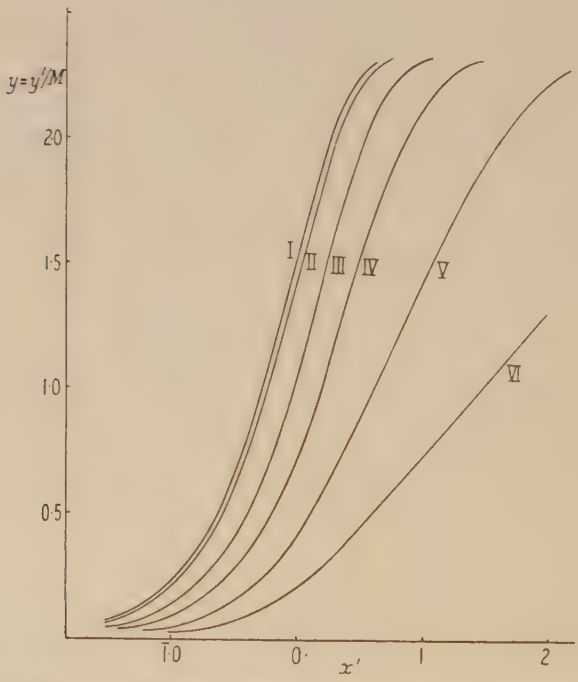

Figure 14. Effect of variation of transmission coefficient α on shape of characteristic curves.
I. Log $\alpha=0$. II. Log $\alpha=\bar{1}{\cdot}9$. III. Log $\alpha=\bar{1}{\cdot}5$.
IV. Log $\alpha=\bar{1}{\cdot}0$. V. Log $\alpha=\bar{2}$. VI. Log $\alpha=\bar{4}$.

constant difference $\log \alpha$ between the abcissae corresponding to the ordinates y_2, y_1 depends only on the penetration of the beam as given by α, the curve may readily be plotted. In this case, however, the $\{D, \log E\}$ curve will approach a maximum density given by

$$-Ar \log \alpha = AdM. \qquad \ldots\ldots(16)$$

When the radiation is fully absorbed ($\alpha = 0$) it is obvious from the equation that the relation between y and x' (or D and $\log E$) will be linear for higher values of E, while at low values the blackening will tend to zero with decreasing E. At very high values of E the phenomenon of solarization occurs. Since this region is of small interest from the point of view of intensity measurements no attempt has been made to take it into account.

In many cases, however, α is not negligible (high-voltage electrons, thin emulsions, etc.), and it becomes of great interest to follow the course of the curve. For very small values of α the term $\exp(-\alpha a \epsilon E)$ will only become appreciably different from unity for large values of $a\epsilon E$ so that a long linear stretch may occur, the extent of which depends on the value of α.

The effect of varying the transmission coefficient α may best be seen in figures 13 and 14. The former shows the variation of the gradient dy/dx' with x' for several values of α, while the latter indicates the effect of variation of α on the shape of the blackening curve.

§ 8. EFFECT OF GRAIN-SIZE VARIATION

The above formula has been calculated on the assumption of unique grain size. In commercial emulsions this does not generally occur, and appropriate corrections should be applied if the grain-size distribution is known. Let $n(a)da$ be the number of grains of areas between a and $a+da$ per unit volume. The formula to be applied may be written

$$\frac{dD}{d \log E} = r \int_{a_1}^{a_2} n(a) a \, da \frac{g-h}{f} (e^{-\alpha a \epsilon E} - e^{-a \epsilon E}), \qquad \ldots\ldots(17)$$

where A has been replaced by its value; a_1 and a_2 indicate the limits of grain size.

A type of grain-size distribution which often occurs in practice is known as the exponential size-distribution and is defined by the equation

$$n(a) = n_0 e^{-\mu a}. \qquad \ldots\ldots(18)$$

Then (17) becomes

$$\frac{dD}{d \log E} = r \int_{a_1}^{a_2} n_0 e^{-\mu a} a \, da \frac{g-h}{f} (e^{-\alpha a \epsilon E} - e^{-a \epsilon E}). \qquad \ldots\ldots(19)$$

In practice we may replace the limits a_1 and a_2 by zero and infinity. This procedure, which has been carried out in other investigations, greatly simplifies the work. Then $\mu = 1/\bar{a}$, where $\bar{a}$ is the mean grain-size and $n_0/\mu = n_0 \bar{a}$ the total number of grains. Equation (19) thus becomes

$$\begin{aligned} \frac{dD}{d \log E} &= n_0 \bar{a} r \bar{a} \frac{g-h}{f} \left[\frac{1}{(1+\alpha \bar{a} \epsilon E)^2} - \frac{1}{(1+\bar{a} \epsilon E)^2} \right] \\ &= \bar{A} r \left[\frac{1}{(1+\alpha \bar{a} \epsilon E)^2} - \frac{1}{(1+\bar{a} \epsilon E)^2} \right], \qquad \ldots\ldots(20) \end{aligned}$$

where $\bar{A}$ has been given the value $n_0\bar{a}\frac{g-h}{f}\bar{a}$ equal to the number of grains per unit volume × mean area × a coefficient depending on their light transmission (the last is assumed independent of grain size and may be taken as approximately equal to unity).

The equation may be integrated :—

$$D=M\bar{A}r\left[\frac{1}{1+\alpha\bar{a}\epsilon E}-\frac{1}{1+\bar{a}\epsilon E}+\log_e(1+\bar{a}\epsilon E)-\log_e(1+\alpha\bar{a}\epsilon E)\right], \quad \ldots\ldots(21)$$

which for low values of $\bar{a}\epsilon E$ becomes

$$D=M\bar{A}r2\bar{a}\epsilon E(1-\alpha),$$

and for high values of $\bar{a}\epsilon E$ tends to the constant value $D=M\bar{A}d$ when all the grains in the emulsion have been developed.

A simple method of obtaining D is by means of the equation

$$y=\log_e(1+z)+1-\frac{1}{1+z}, \quad \ldots\ldots(22)$$

which is shown plotted against $x'=Mx=\log z$ in figure 12. D may be readily obtained by multiplying $M\bar{A}r$ by the difference between two values of $y(y_2, y_1)$ corresponding to the abscissae x' and $(x'+\log\alpha)$, where $x'=\log a\epsilon E$ as before.

The function $y=\log(1+x)+1-\frac{1}{1+x}$ may be compared with the function $y=\int(1-e^{-ex})\,dx$, which fulfils a similar purpose for single-sized grain emulsions. Subject to suitable displacement along the x axis corresponding to an apparent change in $a\epsilon$, the two curves overlap to a remarkable extent within the range generally dealt with (namely, $\log a\epsilon E=\bar{2}$ to $1{\cdot}0$). For values of x' greater than 0·3 the curve nearly coincides with its tangent of slope 1·0. It thus superficially resembles Busé's formula $\log(aE+1)$, but contains extra terms which enable the density to approach a finite limit at high intensities.

For negligible absorption $\alpha=1-d/r$ and formula (21) becomes

$$D=M\bar{A}d\left[1-\frac{1}{(1+a\epsilon E)^2}\right]. \quad \ldots\ldots(23)$$

This may be compared with the corresponding formula $D=MAd(1-e^{-a\epsilon E})$ which obtains for single-sized grain emulsions. For values of $a\epsilon E$ less than 0·8 the only apparent effect is a displacement along the x axis, such that the use of the single-sized grain formula instead of the exponential type leads to values of $a\epsilon E$ which are too great by a factor 1·6.

A somewhat different type of frequency variation with grain size may also be considered, namely, that in which the grain-size distribution is approximately symmetrical about its mean value. For simplicity we shall assume that the number of grains of cross-sectional area between a_1 and (a_1+da) is given by n_0da for values of a_1 between the limits $(\bar{a}\pm a')$, and is zero outside these values. This distribution will tend to accentuate the frequency of the extreme values unduly, and the actual effect of grain-size variation may be expected to be less than that found in this form of distribution.

The mean grain size is $\bar{a}$, and the number of grains $2n_0a'$. The corresponding value of A is therefore

$$2\frac{g-h}{f}n_0a'\,.\,\bar{a}.$$

On integrating as before, it is found that

$$dD/d\log E = Ar\left\{e^{-\alpha\bar{a}\epsilon E}\left(\frac{\sinh\alpha a'\epsilon E}{\alpha a'\epsilon E}\left[1+\frac{1}{\alpha\bar{a}\epsilon E}\right]-\frac{\cosh\alpha a'\epsilon E}{\alpha\bar{a}\epsilon E}\right)\right.$$
$$\left.-e^{-\bar{a}\epsilon E}\left(\frac{\sinh a'\epsilon E}{a'\epsilon E}\left[1+\frac{1}{\bar{a}\epsilon E}\right]-\frac{\cosh a'\epsilon E}{\bar{a}\epsilon E}\right)\right\}. \quad \ldots\ldots(24)$$

The formula is thus similar to that obtained for single-sized grain emulsion, but the exponential factors are modified by correction terms, the value of which it is important to know.

As an example of the effect of grain-size variation, it has been assumed that $a' = \bar{a}/2$, i.e. the grains lie evenly distributed between $\bar{a}/2$ and $3\bar{a}/2$. The error committed by neglecting grain-size variation is about 1 per cent for the extreme variation in grain size suggested.

Table 2

$a\epsilon E=$	0·1	0·2	0·5	1	2	5
Correction factor C	0·987	0·987	0·968	0·957	0·993	1·68
$e^{-a\epsilon E}$ ……	0·905	0·819	$0{\cdot}606_5$	0·368	0·135	0·007
$Ce^{-a\epsilon E}$ ….	0·893	0·808	0·588	0·352	0·134	0·011

In view of the small effect of grain-size variation, we are justified in neglecting it as an approximation, although some of the constants ($A, a\epsilon$, etc.) may be somewhat altered by this assumption.

§ 9. APPROXIMATIONS

In many cases it is possible to use an approximation in the above formula. Thus for low intensities

$$dD/d\log E = Ara\epsilon E(1-\alpha)$$

or

$$D = MAra\epsilon E(1-\alpha), \quad \ldots\ldots(25)$$

representing the well-known linear relation between density and incident radiation at low intensities.

A further approximation valid for higher values of the intensity is given by

$$D = MAra\epsilon E(1-\alpha)\{1-\tfrac{1}{4}a\epsilon E(1+\alpha)+\ \ldots\}, \quad \ldots\ldots(26)$$

showing that the density will tend to fall below the value that would be expected from the linear relation (25).

If the absorption in the emulsion is small ($\alpha \sim 1$) equation (11) leads to

$$D = MAd(1 - e^{-a\epsilon E}), \qquad \ldots\ldots(27)$$

which is Silberstein's formula when no absorption occurs. It is interesting to note that in this approximation beam penetration does not intervene.

As a further approximation, when α is appreciably different from unity, and $a\epsilon E$ is not too great, the formula becomes

$$D = MAd(1 - e^{-a\epsilon E})(1 - d/2r). \qquad \ldots\ldots(28)$$

On the other hand, when absorption in the film is complete, $\alpha = 0$ and

$$dD/d\log E = Ar(1 - e^{-a\epsilon E}), \qquad \ldots\ldots(29)$$

while for absorption which is nearly complete,

$$dD/d\log E = Ar(1 - \alpha a\epsilon E - e^{-a\epsilon E}), \qquad \ldots\ldots(30)$$

showing that the preceding approximation is more likely to be accurate at low values of $a\epsilon E$, the subsequent deviation causing a reduction in the density D.

It will be seen (figure 13) that for values of α in the neighbourhood of unity, there is no accurate linear relation between p and x over an extensive range. Moreover, there is little difference in the shape of the curve between $\alpha = 1$ and $\alpha = 0{\cdot}9$. A good approximation to a linear relation between y and x' does, however, occur for low values of α.

Figure 14 shows clearly the way in which the $\{y, x'\}$ curve depends on α. While no accurate linear slope exists for $\alpha = 1$, a very good approximation may be made, extending over a range of about 0·7 along the x' axis. The intercept on the x' axis of the tangent to the curves increases with decreasing α, while its slope decreases with decreasing α. To compare the slopes it was found preferable to use different ordinate scales so that the various curves should have the same maximum.

§ 10. COMPARISON WITH OTHER FORMULAE

Equation (11) may be compared with other formulae used in relating D to E. The usual formulae are

I. $D = B(1 - e^{-aE})$. Silberstein's quantum formula.
II. $D = C\log(a'E + 1)$. Busé's empirical formula.
III. $dD/d\log E = Ar(e^{-\alpha a\epsilon E} - e^{-a\epsilon E})$.

At low intensities all these assume the form $D = KE$ with $K = Ba$, MCa' and $MAra\epsilon(1 - \alpha)$ respectively.

For higher intensities only III will represent an accurate linear relation between D and $\log E$, and that for small values of α. On the other hand, both I and III will give a good approximation. Formula II will not represent a linear portion, although a good approximation is obtainable for very high values of E. Thus II will only be useful in representing the lower halves of I and III. Moreover, the existence of a maximum density is indicated in I and III, while it is entirely ignored in II. Thus II can only be regarded as an empirical relation valid over a limited range. On the other hand, its simple form may make it of value as a rough approximation for certain purposes.

Formula I can be readily deduced from III for negligible absorption and is thus a special case of the latter when $a \sim 1$. Thus, apart from the greater simplicity of formula I, no extra advantage is to be claimed for it over formula III, while the latter includes those cases of appreciable absorption which cannot be dealt with by I.

§ 11. DOUBLE-COATED EMULSIONS

Double-coated emulsions are often used in x-ray work owing to their greater sensitivity. Formula III can be readily extended to account for their properties. Thus let the absorption in either of the two coatings be negligible, while the transmission coefficient β of the backing of thickness b is appreciable. Formula III for negligible absorption becomes

$$D_1 = MAd(1 - e^{-a\varepsilon E}) \qquad \ldots\ldots(31)$$

and

$$D_2 = MAd(1 - e^{-\beta a\varepsilon E}) \qquad \ldots\ldots(32)$$

for the two coatings. The total density D is then given by

$$D = D_1 + D_2 = 2MAd(1 - e^{-a\varepsilon' E} \cosh a\epsilon'' E),$$

where

$$\epsilon' = \frac{1+\beta}{2}\epsilon \approx \left(1 - \frac{b}{2r}\right)\epsilon$$

and

$$\epsilon'' = \frac{1-\beta}{2}\epsilon \approx \frac{b}{2r}\epsilon$$

$$= (1-\beta)\epsilon'/(1+\beta). \qquad \ldots\ldots(33)$$

Table 3. Values of D/MAd

$a\epsilon E$	Absorption cœfficient $(1-\beta)$			
	0	0·2	0·4	0·6
0·5	0·3935	·3926	·3898	·3814
1·0	·6321	·6299	·6205	·5971
2·0	·8647	·8617	·8474	·8125
4·0	·9817	·9799	·9717	·9476

The effect of absorption is shown in table 3, the photographic density (expressed in terms of the maximum attainable) being calculated for varying absorption and intensity. Even for values of the absorption as great as 40 per cent, the effect on the slope of the $\{D, \log E\}$ curve is very small, and may in most cases be neglected. A small displacement of the curve along the $\log E$ axis, of amount $\log \frac{1+\beta}{2} \approx Mb/2r$, will, however, occur.

§ 12. EFFECT OF DEVELOPMENT

The influence of development on the form of the blackening curve has been investigated by many workers, and it appears to be established that in most cases the influence of the type of developer is small. Developers may be divided into two classes, according to whether the action is based on a reduction of the photographic grain to metallic silver or on a deposition of silver atoms from the developer on the surface of the grain. Bell claims to have shown that the extent of initial linear D/E relation depends on the developer used. This may be due to the difference in the grain size after development.

The increase in photographic density with development time has been shown in many cases to follow the law

$$D = D_\infty(1 - e^{-bt}),$$

where D_∞ is the greatest density possible at the point. Thus the form of the $\{D, \log E\}$ curve does not change with development time, all densities being increased in the same ratio. Chemical fog has been neglected, because it produced a slight general blackening which may be kept small by suitable development conditions.

In view of the increase of densities with development time it is reasonable to assume that each grain follows a similar law of growth $a_+ = a(1 - e^{-bt})$. This assumption is in agreement with the observation that photographic density is proportional to the amount of metallic silver in the developed emulsion. Formula III may be modified to include the effect since $A_t \propto na_t = na(1 - e^{-bt}) \propto A(1 - e^{-bt})$. The grain size a in the exponential must not be varied, since it corresponds to grain size before development, i.e. to the maximum size of the grain.

$$dD/d \log E = A(1 - e^{-bt}) r\{\exp(-\alpha a \epsilon E) - \exp(-a \epsilon E)\}. \qquad \ldots\ldots(34)$$

In most cases, however, this substitution will not be used, it being understood that A is to be calculated from the formula using the grain size a_t after development.

§ 13. APPLICATION OF THE THEORY TO THE PHOTOGRAPHIC ACTION OF ELECTRONS

The fundamental assumption made in the above theory is that the ionization produced by the beams falls off exponentially with increasing penetration, the corresponding range being that of the electron beam in the emulsion. Experiments carried out by the author on the penetration through the glass plate backing of secondary emission due to the electron bombardment of the emulsion (light and x rays) showed it to be small, and, in fact, only appreciable for densities sufficient to cause solarization. It may therefore be assumed that the range of the beam in the emulsion is that of the incident electron beam. This hypothesis is likely to prove satisfactory for electrons of energies somewhat greater than the absorption levels of silver. At these energies the exponential decrease in the ionization of the beam with penetration in the emulsion may also be assumed. Thus ϵ may be considered a constant independent of emulsion thickness, and the previous theory will be expected to apply. To discover whether the theory is, in fact, satisfactory, it has been applied to the experiments of Becker and Kipphan as well as to the author's own results. Weidner's results for slow electrons and some of the work on β rays will also be re-examined in the light of the theory.

(a) *A general form of curve*

Figure 6 shows some of the results obtained by the above observers compared with the theoretical curve deduced above for a single-grain size. Since the grain distribution has not been given, no correction for this can be applied, and some variation in the shape of the curve is probable. Agreement in both shape and extent of the curve is quite satisfactory.

(b) *Penetration of the electrons*

The mean range of the electrons in the emulsion is given by r, the value of which may be determined by the slope of the linear portion of the $\{D, \log E\}$ curve. In the results of Becker and Kipphan for a Matter plate at 22·5 and 84·8 kv. the slopes are respectively 0·51 and 1·04. Unfortunately, the results for grain size and number are not entirely consistent. With the assumption that $A = 2 \times 10^3$ the ranges of the electrons in the emulsion are $2\cdot55 \times 10^{-4}$ and $5\cdot2 \times {}^{-4}$ cm., while if $A = 2\cdot55 \times 10^3$ (corresponding to a maximum density $D = 3$) the corresponding ranges are only 2×10^{-4} and $4\cdot1 \times 10^{-4}$. These values compare satisfactorily with the penetration 3×10^{-4} and 13×10^{-4} in aluminium at the same voltages, when account is taken of the uncertain value of A. Moreover, at 84·8 kv. the absorption in the emulsion is no longer complete; thus if $d/r \gtrsim 1\cdot6$, $\alpha = 0\cdot2$ and the maximum slope is only $0\cdot5Ar$. The range of 80-kv. electrons in the emulsion is then given by $10\cdot4 \times 10^{-4}$ cm. with $A = 2 \times 10^3$, a result in better agreement with the value for Al (13×10^{-4} cm.).

At very low voltages the maximum slope appears to increase with decreasing electron voltage. However, the above theory can no longer be applied, since single electrons are insufficient to render a grain developable.

(c) *Blackening at high electron densities*

Since solarization occurs at high densities, and this phenomenon has not been taken into account in the preceding theory, no exact agreement can be expected. However, an estimate of the extent of the linear region may be made. Thus let the linear region of the slopes be defined as

$$\log(a\epsilon E) \sim 0 \quad \text{and} \quad \log(\alpha a \epsilon E') \sim 0,$$

where E and E' are the incident electron-densities corresponding to the beginning and end of the linear portion. Then $\log E' - \log E = Md/r$. The values of E' calculated from this formula may be compared with those that may be estimated from Becker and Kipphan's paper (figure 8) (22).

Table 4. Value of log E'

Energy	22·5 kv.			28·6 kv.	
r	$2\cdot55 \times 10^{-4}$	$3\cdot5 \times 10^{-4}$		$5\cdot4 \times 10^{-4}$	
log E'	$\bar{7}\cdot8$ (calculated)	$\bar{8}\cdot6$ (calculated)	$\bar{8}\cdot7$ (observed)	$\bar{9}\cdot2$ (calculated)	$\bar{9}\cdot4$ (observed)

Agreement appears to be satisfactory, in view of the large uncertainty in r, due to lack of knowledge of A.

The value of E' is very sensitive to variations in r; an increase of 40 per cent in the latter decreases the former by a factor of over 6.

(d) *Effect of voltage variation*

For low intensities

$$D = Ma\epsilon EAr(1-\alpha).$$

If the penetration r be taken to be very approximately proportional to the voltage of the electron beam and ϵ be assumed independent of voltage, then $D \propto VE$, a result in agreement with the observation made by previous authors that, for a given type of plate, the initial blackening over a considerable range is approximately a functon of the incident energy only.

An application of the preceding theory to Weidner's results is unjustifiable in view of the difference in mechanism between the two types of photographic action. However, it may be of interest to compare the initial sensitivities at various voltages, and these are given in table 1.

At high voltages the absorption within the emulsion is no longer complete and α becomes appreciable. Moreover, ϵ cannot be expected to increase beyond the value unity, which it may be expected to attain in the region 25 to 60 kv. The blackening curve may thus show a saturation effect with increasing voltage, the onset of which is determined by emulsion thickness. Borries and Knoll(25) have shown that this occurs with Agfa Extra Rapid plates at about 70 kv. Ellis and Aston(34) have shown moreover that for higher energies the photographic action decreases approximately as $1/\beta^2$ with increasing electron energy, showing a maximum between 600 $H\rho$ and 1400 $H\rho$ (i.e. about 100 kv.).

(e) *Variation of the coefficient* ϵ

ϵ, being defined as the probability of a grain being developable when transpierced by an electron, may be expected to vary in a somewhat similar fashion to the ionization probability. For very high velocities the latter varies as $1/\beta^2$, but at lower velocities it increases with β. Moreover, it may be expected to show discontinuities at the absorption edges of Br and Ag such as are known to exist with X rays. An estimate of the variation of ϵ may be made from the variation of photographic density D, with the range of the impinging electrons, the intensity of the latter being kept constant. Table 5 has been based on the range as calculated in aluminium. There is a slight variation in ϵ, as might be expected.

An estimate of the magnitude of ϵ may be made from the intercept on the log-E axis. Becker and Kipphan have estimated the constant a' in Busé's equation to be equal to $4{\cdot}54 \times 10^{10}$ (E in coulomb/cm²) for 22·4 kv. electrons. The corresponding value of $a\epsilon$ is $7{\cdot}2 \times 10^{-9}$, while the estimate for a (probably low) is $3{\cdot}4 \times 10^{-9}$, thus $\epsilon \sim 2{\cdot}1$. However, values for ϵ greater than unity are very unlikely, and since the value of grain size is probably too small, we may assume ϵ to be of the order of unity. A similar calculation for 15·5 kv. electrons based on Seitz and Harig's results and with a similar grain size leads to $\epsilon = 0{\cdot}7$.

Table 5. Variation of ϵ with voltage (slight blackening) Matter plate

kv.	Range of electrons in Al ($r\times10^{-4}$ cm.)	Equiv. mass ($m \propto r$)	Density/standard charge, D/E	D/mE or D/rE	Transmission coefficient α (estimated)	$\frac{D}{mE(1-\alpha)}$ or $\frac{D}{rE(1-\alpha)} \propto \epsilon$
40	4·4	1·2 mgm.	6·5*	5·4*	0·015	5·5*
50	6·4	1·73	8	4·6	0·055	4·9
60	8·8	2·39	10	4·15	0·122	4·7
70	11·4	3·08	11·2	3·6	0·19	4·45
80	13·9	3·75	12·4	3·3	0·26	4·46

* Arbitrary units.

(f) *Sensitivity and type of plate*

Of the three types of plate investigated by Becker and Kipphan (Agfa Special Rapid, Matter and Q plates) the first two contain gelatine and are roughly similar in nature, while the last contains very little gelatine and has a light sensitivity about one-seventh of that of the other plates.

For low intensities we have seen that

$$D = MAra\epsilon E(1-\alpha);$$

thus the ratio $D/ME = Ara\epsilon(1-\alpha)$ will serve as a measure of the initial sensitivity of the plate. The difference in sensitivity between different types of plate may be due to different grain size, a different number n per unit volume, greater absorbing power of the emulsion, etc.

A series of values of $Q/D = 1{\cdot}59\times10^{-19}\ E/D$ (Q in coulomb/cm^2) has been given in table 1 for different plate types and voltages.

The Q type of plate having smaller grain size and little gelatine, the number of grains n per unit volume and the absorption in the heavy emulsion will be much higher for the commoner types of plate. Since the linear relation

$$D = MAra\epsilon E(1-\alpha)$$

fails when $a\epsilon E$ becomes appreciable in the second approximation,

$$D = MAra\epsilon E(1-\alpha)\left\{1 - \frac{a\epsilon E}{4}(1+\alpha) + \ldots\right\},$$

this failure only occurs for much higher values of E when a is reduced, and thus a more extensive range will exist in which D and E are proportional. This occurs for Q plates, where the linear relation extends as far as $D = 1{\cdot}5$.

Marton (44) has compared the blackening of a number of plates over the range 50 to 100 kv., using Gaevert Process plates as a standard. At low intensities, x-ray emulsions are more sensitive, while at higher intensities the Process type of emulsion is more sensitive. X-ray emulsions have large grain size a and small grain density n, while the contrary is true for Process emulsions. The second-order term $\frac{1}{4}a\epsilon E(1+\alpha)$ to be subtracted will be most appreciable for the x-ray emulsion, and for suitable values of the constants it is possible for the described variation of sensitivity with exposure to occur.

(g) *Calibration curves at* 50 *kv.*

A series of calibration curves for electrons in the neighbourhood of 50 kv. has been obtained by the author as described above (figure 11). These curves relate $\log I$ to $\log E$ and may thus be taken to be $\{D, \log E\}$ curves with an arbitrary origin. They have been compared with the theoretical curve deduced, using the maximum slope and one point on the non-linear portion. Agreement is very satisfactory, any deviation at low intensities being well within errors due to plate edge-effects, lack of linearity of the {galvanometer deflection, current} characteristic, non-uniform development, etc. Moreover, the graininess in this region is very appreciable.

The patterns on which the calibration curve are based were taken on Ilford Special Rapid plates with different specimens and developed in Kodak Special Developer D 163 for a suitable period (3 to 5 min.).

§ 14. APPLICATION TO β RAYS

Since the absorption of β rays in a thin photographic emulsion is small the expression for the blackening will approach the formula for full transmission,

$$D = MAd(1-e^{-\alpha\epsilon E}). \qquad \ldots\ldots(29)$$

The results of Svedberg and Anderson, obtained for thin layer plates, should satisfy the equation accurately.

Salbach has conducted a series of experiments on β rays using heterogeneous radiation. With Schleussner plates she found the absorption coefficient $\mu = 21\ \text{cm}^{-1}$ Thus absorption within the emulsion is small and to a good degree of approximation the curve may be considered to be of the above form (eqn. (29)). Some of her results are shown in figure 15 and compared with formula III for both $\alpha=1$ and $\alpha=0{\cdot}5$.

Ellis and Wooster found that for homogeneous β radiation, the blackening curve can be represented by Busé's formula

$$D = C\log\left(\frac{It}{T}+1\right).$$

As has been shown above, this formula represents the theoretical formulae I and III satisfactorily over a certain range. In particular, for negligible absorption in the emulsion it can be shown that C is to be replaced by Ad (equal to $n_0a(1-e^{-bt})d$) and thus depends on the type of emulsion and its development. $1/T$, however, corresponds to $a\epsilon$ and is thus independent of development time, but does depend, for a given grain size a, on the energy of the incident particles through the coefficient ϵ. This is in entire agreement with Ellis and Wooster's results, since they found that, while C depends on development time, T is determined by the particle energy. In their experiments, Ilford x-ray emulsions were used, so that the absorption might become appreciable, when the above values for C and T might be slightly modified, but the agreement with theoretical predictions would be unaffected.

In a subsequent investigation, Ellis and Aston showed that the photographic action of β rays decreased more rapidly than the ionization per cm. with rising energies. This may be due to appreciable absorption in the emulsion, but in any case the meaning of photographic action must be well defined, since a measurement of photographic action by means of the blackening produced is only justifiable at low blackening values.

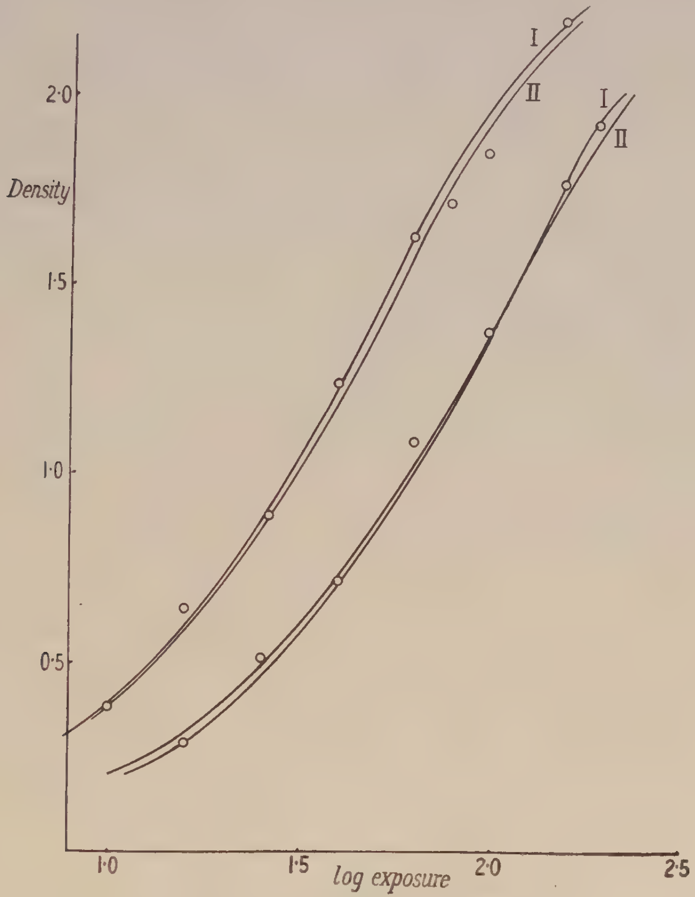

Figure 15. Blackening by β rays.

o Experimental values (Salbach).
I. Theoretical curve assuming negligible absorption.
II. Theoretical curve assuming 50 per cent absorption.

§ 15. APPLICATION TO X RAYS

The penetration of x-ray photons in the photographic emulsion may be taken to obey the exponential law assumed in the theoretical investigation and, subject to reservations regarding grain-size variations, the formula deduced above may be expected to hold for their action on photographic emulsions.

Early work by Friedrich and Koch and by Glocker and Traub indicated that the form of the blackening curve was independent of the wave-length of the incident radiation. On the other hand, to bring blackening curves into coincidence it might be necessary to alter the intensities in a suitable ratio depending on the wave-length. These results are in complete agreement with the preceding theory, since with negligible absorption ($\alpha \sim 1$) the maximum slope $dD/d \log E = Ade^{-1}$ does not depend on the wave-length. The factor ϵ, however, will so depend, but its effect may be compensated by multiplying E by a suitable factor depending on the variation in ϵ, i.e. depending on λ.

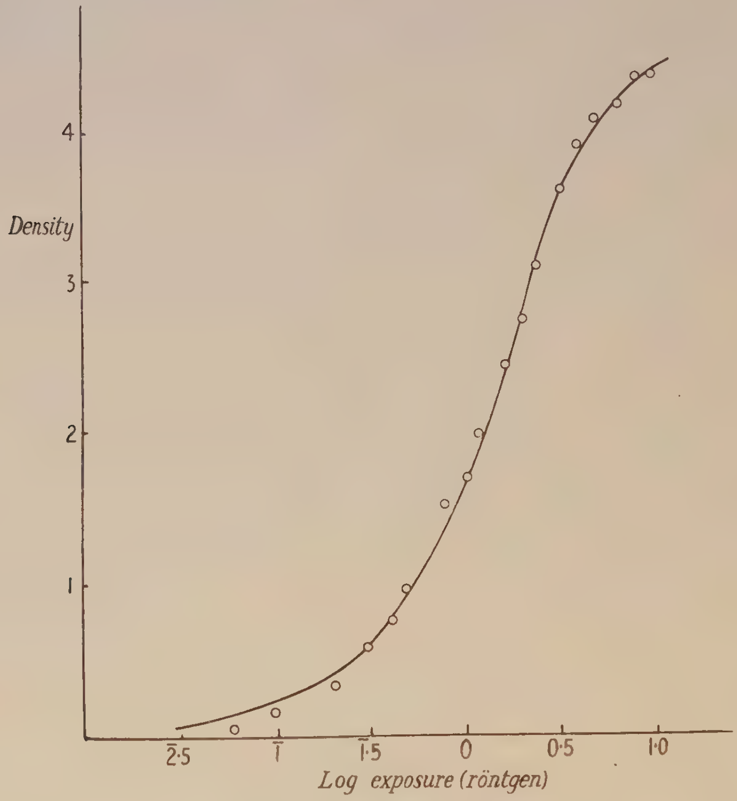

Figure 16. Blackening by x rays.

o Experimental values (Bell).

——— Theoretical curve $D = 4{\cdot}41(1 - e^{-0{\cdot}479E})$.

The blackening curves obtained by Glocker and Traub have been compared with formula III for the two limiting cases ($\alpha = 1$ and $\alpha = 0$), the difference between the two being small over most of the range considered. It is seen that there is no systematic variation with wave-length apparent in the figure. Bouwers's results are similar in nature in so far as the variation with wave-length is concerned, systematic deviations from the simpler Busé formula $D = C \log(at + 1)$

being in the direction of formulae III and I. Agreement with the latter as proposed by Blau and Altenburger is satisfactory if the coefficient a is allowed to vary with λ.

Silberstein and Trivelli have shown that the formula $p = 1 - e^{-an}$, which gives the probability p of a grain of area a becoming developable when subjected to an incident radiation of n photons per unit area, and which was unsuccessful for light, holds satisfactorily for X-ray photons and thin emulsions. Formula III is then substantiated, since, by the conditions of the experiment, $\alpha = 1$, when III simplifies to I, a modified form of the above expression.

Bell, in his investigation of the photographic action of X rays, used four different types of plate, three being double-coated emulsions of the usual X-ray type, while the fourth was an ordinary plate.

The blackening curve of the ordinary plate has been compared in figure 16 with the theoretical formula where $\alpha = 1$, so that with suitable values of the constants $D = 4{\cdot}41\,(1 - e^{0{\cdot}479E})$. Agreement with the theoretical formula remains satisfactory even at high intensities in the neighbourhood of saturation, whereas the empirical Busé formula indicates erroneously a steady rise in the density.

The intercept of the asymptote on the log E axis is at 0·35 röntgen, while the corresponding value of $a\epsilon E$ from the theoretical curve ($\alpha = 1$) is 0·166 (E being expressed in photons/cm^2). A very rough estimate of the number of photons in a beam of 1 röntgen intensity, based on the absorption coefficient in air, leads to a value of 5×10^9 photons. The corresponding value of $a\epsilon$ is $9{\cdot}6 \times 10^{-11}$ from the constants in the above and $9{\cdot}5 \times 10^{-11}$ from the intercept.

(a) *Blackening at low and medium intensities*

Unlike Bouwers, Bell found that at low intensities blackening was proportional to the incident radiation up to fairly high values of D, depending naturally on the type of plate. The corresponding approximation is given by

$$D = MAra\epsilon E(1-\alpha)\{1 - \tfrac{1}{4}a\epsilon E(1+\alpha) + \,.\,.\,.\}$$

and the linear relation between D and E may thus be expected to break down when the correction term $-\tfrac{1}{4}a\epsilon E(1+\alpha)$ becomes appreciable. This will occur at higher values of E for smaller values of grain size. Thus the ordinary plate should have a greater linear range than the coarser grained X-ray plates, this being in agreement with Bell's observation. Moreover, since 0·35 röntgen has been seen to correspond to $a\epsilon E = 0{\cdot}166$, a 5 per-cent deviation from the linearity may be estimated to occur at about 0·21 röntgen. The figure found by Bell is of about this value.

At low intensities we have seen that

$$dD/d\log E = a\epsilon EAr(1-\alpha),$$

and Bell has given the value of E corresponding to $dD/d\log E = 0{\cdot}2$. An estimate of the quantity $a\epsilon Ar(1-\alpha)$ or $a\epsilon Ad$ (since $(1-\alpha)r$ is nearly equal to d for penetrating radiation) may thus be made. By assuming suitable values for Ad from the maximum attainable density (MAd), an estimate of $a\epsilon$ may then be deduced.

The values of $a\epsilon Ad$ obtained are compared with those obtained from a more direct comparison of two of the blackening curves with the theoretical curves. As before, it has been assumed that 1 röntgen $=5\times10^9$ photons.

Table 6

Emulsion	Exposure in röntgen	$a\epsilon Ar(1-\alpha)$ $\sim a\epsilon Ad$ from curve slope	Approx. value of $a\epsilon$		$a\epsilon Ad$ from equation to curve
			$(Ad=10)$	$(Ad=6{\cdot}9)$	
Double-coated A	0·012	$3{\cdot}3\times10^{-9}$	$3{\cdot}3\times10^{-10}$	$4{\cdot}8\times10^{-10}$	$3{\cdot}6\times10^{-9}$
B	0·015	$2{\cdot}7\times10^{-9}$	$2{\cdot}7\times10^{-10}$	$3{\cdot}6\times10^{-10}$	$2{\cdot}9\times10^{-9}$
C	0·024	$1{\cdot}6\times10^{-9}$	$1{\cdot}6\times10^{-10}$		
Ordinary A	0·045	$0{\cdot}9\times10^{-9}$	$0{\cdot}9\times10^{-10}$		

The estimate of $a\epsilon$ for the ordinary plate (9×10^{-11} cm^2) based on the form of the curve at low and high intensities is thus in good agreement with that found from the slope of the curve (medium intensities) ($9{\cdot}5\times10^{-11}$ cm^2), and is a strong argument in favour of the general correctness of the shape of the curve deduced in the theoretical argument and of the meaning to be attached to the constants.

The behaviour of emulsions A and B at low intensities has also been compared with that predicted by formula III for the values of the transmission coefficient α (figure 17). The corresponding equations are

$$D=2{\cdot}99_3\,(1-e^{-2{\cdot}63E})\quad \text{if } \alpha=1 \text{ for emulsion A,}$$
$$D\sim2{\cdot}99_3\,(1-e^{-2{\cdot}95E'})^*\quad \text{if } \alpha=0{\cdot}9 \text{ for emulsion A,}$$
$$D=3{\cdot}22_4\,(1-e^{-1{\cdot}95E})\quad \text{if } \alpha=1 \text{ for emulsion B.}$$

From these figures and assuming as before that 1 röntgen corresponds to 5×10^9 photons under the conditions of the experiment, the values of $a\epsilon$ and Ad for the two types of plate may be deduced:—

$a\epsilon=5{\cdot}26\times10^{-10}$ $(\alpha=1)$ or $5{\cdot}9\times10^{-10}$ $(\alpha=0{\cdot}9)$ for emulsion A.

$a\epsilon=3{\cdot}90\times10^{-10}$ $(\alpha=1)$ for emulsion B. The corresponding values of Ad are 6·9 and 7·5.

The values of $a\epsilon Ad$ for the two types of plate based on the shape of the curve at medium intensities can be calculated and are in satisfactory agreement with those deduced from the slope at a point, the values being shown in columns 4, 5 and 6 of table 6.

For the single-coated X-ray plate whose characteristic is shown in figure 9 (19), a similar value of $a\epsilon$ may be expected ($a\epsilon\sim3$ or 4×10^{-10} cm^2). The value obtained from the blackening curve is $2{\cdot}6\times10^{-10}$ if $\alpha=1$ and $3{\cdot}2\times10^{-10}$ if $\alpha=0{\cdot}9$. The formula $D=4{\cdot}50\,(1-e^{-1{\cdot}29E})$ suggested by Bell also leads to $a\epsilon=2{\cdot}6\times10^{-10}$ and $Ad=10{\cdot}4$. The latter value can be checked since it corresponds to a maximum density of 4·5, this being a reasonable approximation to the curve as shown in figure 9 (19).

It thus appears that both as regards the shape of the characteristic curves, and the meaning to be attached to the constants which determine them, close agreement is obtained by the use of the theoretical formula deduced above.

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* Approximation for D instead of accurate infinite expansion. $E'=EK$, where K corrects for displacement of the x-axis between $\alpha=1$ and $\alpha=0{\cdot}9$ curves.

(b) *Variation in wave-length: heterogeneous radiation*

The effect of wave-length variation for homogeneous radiation has already been discussed by Friedrich and Koch as well as Glocker and Traub. The effect of heterogeneous radiation was studied by Bell. He showed that the maximum sensitivity of photographic plates occurs for radiation having a half-value layer of about 0·5 mm. in Cu. This may be regarded either as selective absorption in each grain at about 0·5 A., or as being due to variation of both r and ϵ with wave-length, the resultant density function showing a maximum.

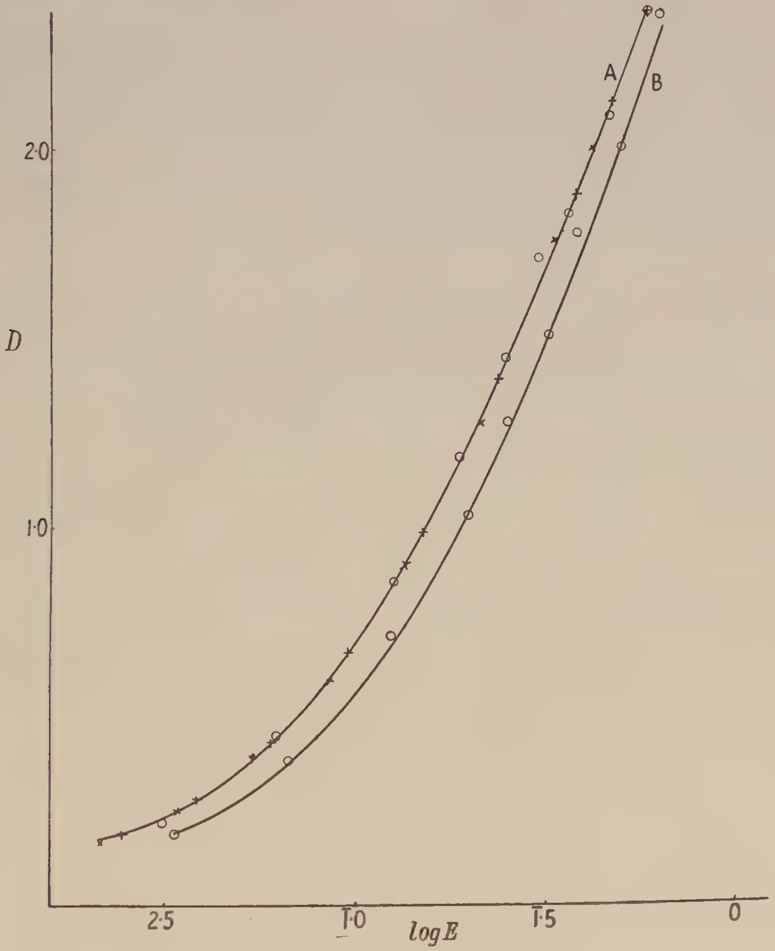

Figure 17. Blackening by x rays.
o Experimental values (Bell).
——— Theoretical curve ($\alpha = 1\cdot0$). + ($\alpha = 1\cdot0$). × ($\alpha = 0\cdot9$).

As a first approximation, formula I, which applies for negligible absorption,

$$D = MAd(1 - e^{-a\epsilon E}),$$

might be expected to be applicable for heterogeneous radiation of 80 kv., 120 kv. and 175 kv. With increasing voltage it is found that if the intensities are increased in the ratio 1 : 1·9 : 3·2, the resultant blackening curves coincide. This may be taken as due to a corresponding decrease in the value of ϵ with increasing photon energy, the quantity $a\epsilon E$ then remaining constant. An estimate of the variation of ϵ with wave-length may be made by assuming it to be analogous to the probability of ionization. From the half-value layers (0·24, 1 and 2 mm. Cu) corresponding to the exciting voltages (80, 120 and 175 kv.) the equivalent wave-lengths 0·267 A., 0·152 A. and 0·110 A. may be calculated. Assuming the same energy to be given to each ion pair, the ionization probabilities must be as 3·44 : 1·47 : 1. The values of ϵ calculated from the intensity ratio are 3·2 : 1·68 : 1. It thus appears that the variation of ϵ closely follows the probability of ionization.

§ 16. SUMMARY

A formula for the blackening of photographic plates by high-energy radiation and taking into account the effect of absorption has been deduced. Certain simplifying assumptions (exponential absorption in the emulsion, type of reaction within the grain) are made, and the corresponding formula obtained is such that all the coefficients occurring in it may be given simple physical meanings (e.g. mean grain size, density, range of radiation, etc.). To determine the validity of the expression deduced, its predictions are compared with experimental results. Unfortunately, exact quantitative comparison is not always possible, since information necessary for this purpose is sometimes lacking. Nevertheless, in view of the many possible points of comparison possible, agreement is very satisfactory, the following points being especially noteworthy.

The general shape of the curve for both electrons and X rays agrees well with those found experimentally, the variation in the forms being explained in terms of the difference in the absorption of the incident beams. An extensive portion of the {blackening, logarithm of exposure} curve is found to be accurately linear only when full absorption of the incident beam occurs. From the shapes of the curve the grain characteristics of the emulsions are deduced, good agreement being obtained with the experimental values. In particular, values obtained from the shape of the curve at a point and from the maximum slope, are in good agreement, thus affording an additional argument for the general correctness of the theory. The range of the beam in the emulsion and the maximum density may also be satisfactorily deduced.

The variation of the shape of the blackening curve with particle energy for electrons, and its apparent constancy for X rays are explained as being due to the more complete absorption of the radiation in the former case. This explanation also applies to observed maximum sensitivity of plates for electrons of about 100 kv. energy. The more extensive linear relation between blackening and exposure for Q type plates (electrons) and ordinary type plates (x rays) is also predicted and the relative sensitivities of x-ray and Process plates for electrons are investigated.

A more detailed analysis would involve a fuller knowledge of the grain characteristics of the emulsion, as well as of the precise mechanism of the individual reactions. In view of the good agreement obtained with the simpler assumptions, it appears that these are generally sufficient to lead to very satisfactory results.

§ 17. CONCLUSION

In the investigation of the response of a photographic emulsion to incident radiation, the absorption of the beam, if appreciable, plays a considerable part. A theory based on an exponential type of absorption leads to a formula representing the observed phenomena very closely. Even when the absorption is small, however, as in the case of x rays, the formula proves effective in accounting for certain apparently disconnected observations. Any theory of the blackening of photographic plates should thus include some account of the absorption of the beam within the emulsion.

Since the formula contains factors depending on the grain properties of the emulsion, its use may provide a simpler means of estimating the latter. In particular the extent, position, and slope of the linear portion of the {blackening, logarithm of exposure} curve may be estimated from the grain characteristics of the emulsion and from its absorptive power. It has been shown that the slope of this linear portion depends on the range of the incident radiation for high-energy particles, and this is also probable for low-energy particles.

The variation in the photographic action of high- and low-energy radiation can be readily explained in terms of the difference between the single and multiple types of reaction in the sensitive grain. Although the formula has only been deduced for particles of high energy, its extension to lower energies should not prove difficult. Any comprehensive formula, however, must be such as to lead to blackening curves in the intermediate region between the two types of reaction, which exhibit characteristic inflexions absent from the more usual blackening curves.

§ 18. ACKNOWLEDGEMENTS

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NOTE ON THE COMPOSITION OF ALLOYS WITH COMPOSITION NEAR Al_2Cu

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ABSTRACT. The note gives a theoretical explanation of the experimental finding (Stockdale (1)) that there is no stable single-phase copper-aluminium alloy with the exact composition Al_2Cu, although there is a small range of compositions lying a little to the aluminium-rich side of this for which such single-phase alloys exist.

The explanation is based on free-energy considerations. Available data (2) for the heats of formation of these θ-phase alloys and those of the neighbouring phases in the copper-aluminium phase diagram suggest that the free-energy curves for θ-phase alloys (free energy versus composition at a given temperature) will lie at a higher level than those for the neighbouring phase on the copper-rich side (the η-phase). Consequently common tangents between corresponding free-energy curves for the η- and θ-phases will touch the curves for the latter phase on the aluminium-rich side of their minima. The points of contact give the first boundary of the θ-phase. The assumption (Bradley and Phragmén (1)) that the excess aluminium (above the 2:1 atomic ratio) in a θ-phase alloy replaces copper atoms in an otherwise "ordered" phase allows us to calculate the shape of the free-energy curves for the θ-phase in the neighbourhood of their minima, and to show that this phase will begin with excess aluminium of the amounts found by Stockdale if the energy of substitution of excess aluminium in a θ-phase alloy is approximately 21,000 $\boldsymbol{k}$.

According to Stockdale (1) an aluminium-copper alloy with the exact stoichiometric ratio of two atoms of aluminium to one atom of copper consists always of a mixture of two distinct phases. There is no stable single phase with the composition Al_2Cu, at any rate at temperatures above 400° c. The θ-phase of aluminium-copper alloys, which previously was thought to include the compound Al_2Cu, is shown by Stockdale to extend over a narrow range of compositions all lying on the aluminium-rich side of the 2:1 ratio.

In the correspondence on Stockdale's paper (1), Bradley and Phragmén both emphasize that x-ray analysis of this θ-phase reveals that its lattice structure is of the type PQ_2, with twice as many Q positions in the lattice as P positions. Therefore the most likely interpretation of Stockdale's result, advanced by Bradley and by Phragmén, is that all the Q positions in the lattice are occupied by aluminium atoms, the great majority of the P positions by copper atoms, and the remainder of the P positions by (excess) aluminium atoms.

At first sight, perhaps, this is surprising. For the narrowness of the θ-phase suggests that an appreciable amount of energy is required to replace a copper atom by an aluminium atom, and we might expect that the composition of minimum energy, in which there is no excess aluminium, would lie within the range of that phase. But we must not forget that a phase diagram is determined by the free energies of the neighbouring phases, as well as the phase under consideration. In this note we shall try to find an explanation of the position of the θ-phase in Stockdale's phase diagram by considering how these free energies are likely to vary with the composition.

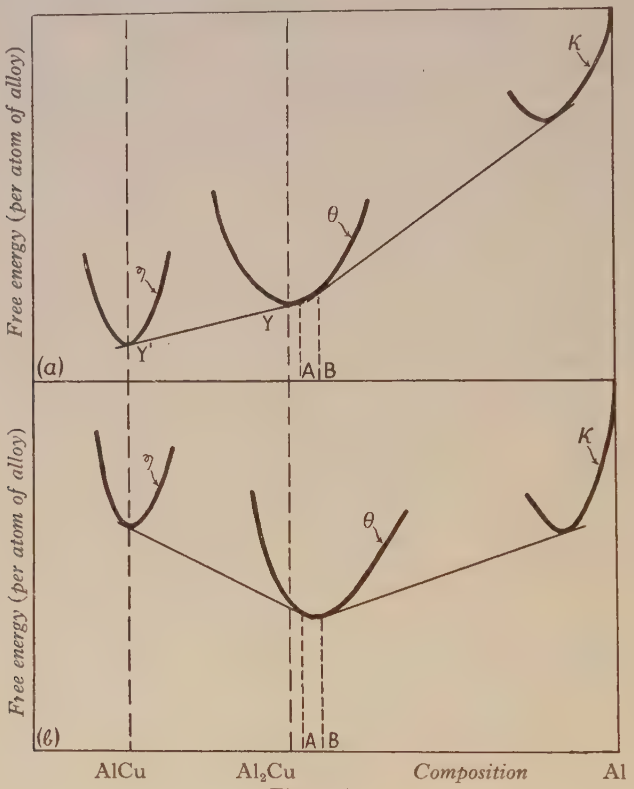
Free energy (per atom of alloy)
K
θ
η
Y
Y'
(a)
A
B
Free energy (per atom of alloy)
K
η
θ
(b)
A
B
AlCu
Al₂Cu
Composition
Al

Figure 1.

The θ-phase of aluminium-copper alloys is flanked in the phase diagram by the η- and κ-phases, which include respectively the alloy AlCu and pure aluminium. They include also, of course, solid solutions of aluminium and copper having the same crystal structures as those substances. If the free energy of formation of these solid solutions (per atom and at a given temperature) be plotted against the percentage of aluminium therein, we get a free-energy curve for each of the η-, θ- and κ-phases. The points of contact of common tangents between these curves give directly the limits of solubility of aluminium in the θ-phase, at the given temperature. Since Al_2Cu does not exist without excess aluminium, both of these points of contact must lie on the aluminium-rich side of the abscissa corresponding to the 2 : 1 atomic ratio.

Figure 1 illustrates schematically two quite distinct ways, (*a*) and (*b*), in which this can arise. In each case the range of composition, AB, of the θ-phase excludes the compound Al_2Cu, including only solutions richer in aluminium. In (*a*), however, the reason for this is that the free-energy curve for the η-phase lies at a lower level than that for the θ-phase, while in (*b*) it is because the minimum of the latter curve itself belongs to an aluminium-rich alloy. We have to decide in which, if either, of these ways the non-existence of the compound Al_2Cu is correctly to be explained.*

Examining first the possibility of an explanation of type (*a*), we observe that the difference in depth of the minima of the free-energy curves for the η- and θ-phases is approximately equal to the difference between the heats of formation (per atom of alloy) of AlCu and the alloy usually denoted by Al_2Cu. The correction on account of the entropy change is negligible at temperatures well removed from the order-disorder transformation temperature for the η-phase (approx. 550° C.), and at higher temperatures may amount to about 1 kcal. per N atoms, N being Avagadro's number. The heats of formation of AlCu and "Al_2Cu" are given by Bichowski and Rossini$^{(2)}$ as 68 and 84 kcal. per gram molecule respectively. Since a gram molecule of AlCu contains $2N$ atoms while a gram molecule of "Al_2Cu" contains $3N$ atoms, the heats of formation per N atoms of these alloys are respectively 34 and 28 kcal. These values relate, strictly, to a temperature of 18° C., but the corrections to be made to them when the temperature is, say, 500° C. can be estimated from specific-heat data and are quite negligible. We conclude that the minimum of the free-energy curve for the η-phase is at a lower level (by 6 kcal. per N atoms of alloy) than the minimum of the free-energy curve for the θ-phase, and therefore that the non-existence of the compound Al_2Cu is explicable in the way which we have called (*a*). This explanation can fail only if the minimum of the free-energy curve for the θ-phase lies on the copper-rich side of the abscissa for Al_2Cu, and then only if the slope of the free-energy curve where it crosses this abscissa is greater than that of the common tangent Y′Y (figure 1). We shall now consider the probable shape of this free-energy curve in the neighbourhood of its minimum.

Suppose that there are $N(\frac{1}{3}+x)$ aluminium atoms and $N(\frac{1}{3}-x)$ copper atoms in a θ-phase alloy. Then if $x>0$ the percentage excess aluminium is $100x$, while if $x<0$ the percentage excess copper is $100\,|x|$. Let E_1 and E_2 be the energies of substitution in a θ-phase alloy of excess aluminium and copper atoms respectively.† These energies have apparently not been determined experimentally. We can deduce, however, from the lack of any order-disorder transition in θ-phase alloys (which are "ordered" at all temperatures up to their melting temperatures) that E_1+E_2 is large compared with $\boldsymbol{k}T$, where $\boldsymbol{k}$ is Boltzmann's constant. Actually we shall see below that Stockdale's results require $E_1/\boldsymbol{k}T \simeq 30$, which confirms this conclusion. It follows, for not too small x,* that if there is an excess of aluminium it will occupy copper (P) sites while if there is an excess of copper it will occupy aluminium (Q) sites, in agreement with Bradley and Phragmén's interpretation of the P and Q sites of a θ-phase lattice. Since we are interested only in small excess percentages of copper or aluminium we may suppose that these extra atoms are distributed randomly among the other species of atom, which they are replacing. On these assumptions it is easy to write down the entropy of mixing. There are $\frac{2}{3}N$ P-sites and $\frac{1}{3}N$ Q-sites, and so we have

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* In constructing free-energy diagrams like those of figure 1, it is immaterial whether we employ free energies of formation from isolated atoms or free energies of formation from the constituent metals in the solid state; in this note we shall, throughout, use the latter convention.

† i.e. E_1 is the energy required to add an excess aluminium atom, drawn from pure aluminium, the copper atom which it displaces being transferred to a pure copper phase.

$$\left.\begin{aligned} \Delta S &= \boldsymbol{k}\,.\log\frac{(\frac{1}{3}N)\,!}{(\frac{1}{3}N-Nx)\,!\,(Nx)\,!} \qquad \text{when } x \geqslant 0, \\ \Delta S &= \boldsymbol{k}\,.\log\frac{(\frac{2}{3}N)\,!}{(\frac{2}{3}N+Nx)\,!\,(-Nx)\,!} \qquad \text{when } x \leqslant 0. \end{aligned}\right\} \qquad \ldots\ldots(1).$$

When the amount of excess (or deficit) aluminium is small we may suppose that the energy changes (ΔE) are proportional thereto. The free energy of formation, per atom of alloy, is therefore given by

$$\left.\begin{aligned} f &= E_1x-\boldsymbol{k}T[\tfrac{1}{3}\log\tfrac{1}{3}-(\tfrac{1}{3}-x)\log(\tfrac{1}{3}-x)-x\log x] \quad \text{when } x \geqslant 0, \\ f &= -E_2x-\boldsymbol{k}T[\tfrac{2}{3}\log\tfrac{2}{3}-(\tfrac{2}{3}+x)\log(\tfrac{2}{3}+x)+x\log(-x)] \quad \text{when } x \leqslant 0. \end{aligned}\right\} \ldots\ldots(2).$$

In the ranges of x for which these formulae are valid, i.e. except for values of x which are very small, in fact smaller by several powers of 10 than those corresponding to the commencement of the θ-phase in Stockdale's diagram (for which $10^{-3} < x < 10^{-2}$), $\left(\frac{\partial f}{\partial x}\right)_T$ is a steadily increasing function of x. For $x \geqslant 0$ we have

$$\left(\frac{\partial f}{\partial x}\right)_T = E_1 - \boldsymbol{k}T\log\left(\frac{1}{3x}-1\right) \qquad \ldots\ldots(3).$$

The θ-phase will commence at that value of x for which

$$\left(\frac{\partial f}{\partial x}\right)_T = m,$$

where m is the gradient of the common tangent Y′Y. For that value of x at which the θ-phase begins, we have therefore

$$\boldsymbol{k}T\,.\log\left(\frac{1}{3x}-1\right) = E_1 - m \qquad \ldots\ldots(4),$$

which shows that if the θ-phase is to commence with an excess of aluminium (above the 2:1 atomic ratio) then the energy required to replace a copper by an aluminium atom therein has to be rather greater than the slope of the common tangent between the η- and θ-phases. We have already estimated that this line, Y′Y, has a slope corresponding to an energy change of about 6 kcal. between AlCu and "Al_2Cu", i.e. for a change of 16·7 in the percentage proportion of aluminium in the alloy. This gives

$$m \simeq 18{,}000\ \boldsymbol{k}$$

(where $\boldsymbol{k}$ is Boltzmann's constant).

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* This distribution of atoms among the sites will be correct to an accuracy of 1 : 100 $x > 10\ .\ \exp.\ [(E_1 + E_2)/2\boldsymbol{k}T]$, which we shall find is probably of the order of 10^{-7}, and so negligibly small.

If E_1 and m were known accurately and were independent of T, equation (4) would enable us to construct the left-hand boundary of the θ-phase in a phase diagram. As it is, a mild check on the explanation put forward in this note for the position of that boundary is afforded by estimating $(E_1 - m)$ by means of equation (4) from several points on the left-hand boundary of the θ-phase in Stockdale's phase diagram. This boundary indicates that the θ-phase begins with 0·3 ($x = 3 \times 10^{-3}$) excess percentage of aluminium when $T = 400°$ C., and with a greater excess percentage at high temperatures. When temperatures corresponding to $x = 3, 4, \ldots 8 \times 10^{-3}$ are obtained from Stockdale's graph and substituted in equation (4) we find six values for $E_1 - m$, and they all lie between 3100 $\boldsymbol{k}$ and 3400 $\boldsymbol{k}$: the relatively small spread suggests that this explanation is on the right lines. Since $m \approx 18{,}000\,\boldsymbol{k}$ we deduce that $E_1 \approx 21{,}000\,\boldsymbol{k}$.

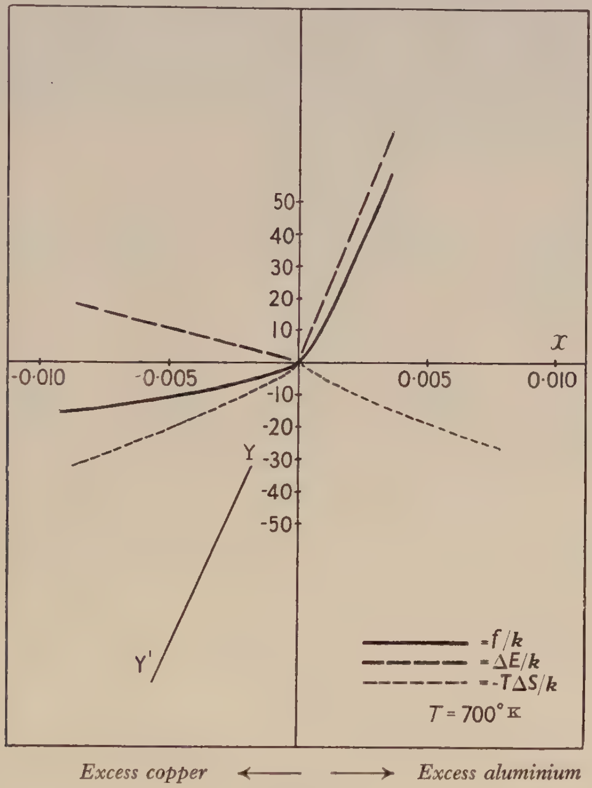

Figure 2.

Figure 2 shows f (equation (2)), as a function of x for $T = 700°$ K. It is the sum of ΔE and $-T\Delta S$, and its actual shape will depend entirely upon the magnitudes of E_1 and E_2. The figure is drawn for $E_1 = 20{,}000\,\boldsymbol{k}$, $E_2 = 2000\,\boldsymbol{k}$. We have shown that E_1 has a value of this order of magnitude, and though we do not know the true value of E_2 we would expect E_2 to be less than E_1, since the atomic radius of aluminium in solid solutions is greater than that of copper⁽³⁾. The figure illustrates the point that though the minimum of the free-energy curve for the θ-phase may lie on the copper-rich side of the abscissa corresponding to Al_2Cu, this does not matter if the common tangent Y′Y is sufficiently steep. It is clear also that, whatever the values of E_1 and E_2, the figure rules out an explanation on the lines of figure 1 (*b*).

We conclude therefore that the non-existence, at temperatures at the level investigated by Stockdale, of the alloy Al_2Cu is due principally to the free energy of formation per atom of AlCu being greater than that of " Al_2Cu ". It also appears from the available data that the energy of substitution of excess aluminium in a θ-phase alloy is approximately equal to 21,000 $\boldsymbol{k}$.

I should like to thank Professor N. F. Mott, F.R.S., for pointing out to me the anomalous position of the θ-phase in the phase diagram of copper-aluminium alloys, and both him and Dr. C. A. Coulson for helpful interest in the writing of this note.

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DISCUSSION

Dr. H. Lipson. The calculations of Mr. Rushbrooke support the view, which I have held for some time, that the many cases in alloy systems of the non-occurrence of the stoichiometric composition within the phase field is a general phenomenon explainable by his construction in figure 2. There are certain general principles which arise from this, if one makes the plausible assumption that the free-energy curves have their minima at the stoichiometric compositions.

1. Since only one free-energy curve can have its minimum below the minima of the other curves, only *one* phase field should include the stoichiometric composition.

2. In the case where two minima have exactly the same value, the limits of the phase fields should occur exactly at the stoichiometric compositions. In effect, this means that, as the temperature varies, the composition of lowest free energy may change from one phase field to another.

3. The phase fields which lie nearer to one component than does that of the lowest free energy should be displaced towards that component.

An examination of the binary diagrams in Hansen's *Der Aufbau der Zweistofflegierungen* shows that there are very few systems which can provide a test for these generalisations. In any case, the degree of support given can only be regarded as a measure of the plausibility of the assumption mentioned above.

A COMPLEX BAND-SPECTRUM ASSOCIATED WITH AN OXIDE OF SILICON

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Received 25 *June* 1940

ABSTRACT. A band system of complex structure, ascribed to an oxide of silicon, possibly SiO_2, is described. The source was a heavy-current discharge tube with a quartz constriction. The spectrum was photographed under high dispersion and appears to be an extension of one previously reported by Cameron.

§ 1. SOURCE AND APPEARANCE OF THE SPECTRUM

DURING experiments with a heavy-current hydrogen discharge tube of a type⁽¹⁾ in which the discharge was made to pass through a Vitreosil quartz constriction, an unfamiliar band-system in the violet region was observed by means of a visual spectroscope. A photograph of the system was taken on an Ilford Zenith plate in the first order of a six-metre concave grating with 11,000 lines to the inch giving a dispersion of 4 A./mm. With a secondary current of 1–1½ amp. at 2500 volts the exposures occupied about ten minutes. Plates were also taken on a Hilger E 2 quartz prism spectrograph.

The most conspicuous feature of the spectrum is the complicated structure in the region λλ 4280–4220, comprising several rectangular "blocks", a number of bands degraded to the red and a few others degraded in the opposite direction.

The rest of the spectrum consists mainly of bands degraded to the red superimposed on a complex background the intensity of which rises periodically to maxima. It is difficult to give an adequate description of the system by means of a table of wave-lengths: besides the general complexity, the heads are not always sharply defined. The measurements below are not in all cases reliable to less than 0·5 A., but for a number of the features listed the accuracy is probably rather better than this figure.

§ 2. IDENTIFICATION OF THE EMITTING MOLECULE

The system is believed to be due to an oxide of silicon excited in the intensely hot quartz constriction in the discharge tube. The bands were again obtained from a fresh tube in which a different specimen of quartz was used for the constriction. The quartz continued to glow bright red for several seconds after the

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Table

Author				Cameron	
λ	ν	*I*		λ	*I*
4646	21518	0	M	4648·2	1
				4637·1	1
4587	21795	1	M	4585·0	1
				4577·1	2
4528	22079	2	M		
4525·7	22090	2	R	4525·6	3
				4516·1	3
4476	22335	1	M		
4474·0	22345	1	R	4474·6	2
4468·5	22373	4	R		
4466·6	22382	3	R	4466·3	4
				4454·2	3
4417·3	22632	3	R	4417·5	2
4410	22669	4	M	4408·9	5
4407	22685	4	? R		
4392·6	22759	2	R	4393·4	2
4360·0	22929	1	V		
4340	23035	10	M		
4310·5	23193	2	R		
4296·8	23267	2	R		
4288·0	23314	3	R		
4280–4220		10	M		
4277·6	23371	4	Bl		
4274·5	23388				
4273·4	23394	4	Bl		
4269·6	23415				
4262·8	23452	6	R		
4256·6	23486	8	R		
4254·4	23498	7	R		
4252·4	23510	6	Bl		
4243·4	23559				
4241·8	23568	10	Bl		
4239·6	23581				
4235·5	23603	10	V		
4228·5	23642	9	V		
4280–4220		10	M		
4071·7	24553	1	R		
4069	24569	1	M		
4022	24856	3	M		
4019·4	24872	3	R		
3972	25169	4	M		
3922	25490	4	M		
3835	26068	6	M		
3832·9	26083	6	R		
3798	26322	2	M		
3791·4	26368	2	R		
3776·9	26469	3	R		
3713·5	26921	2	R		
3672·8	27219	1	V		

R: Band degraded to the red.
V: Band degraded to the violet.
Bl: Rectangular " block ".
M: Maximum of intensity.

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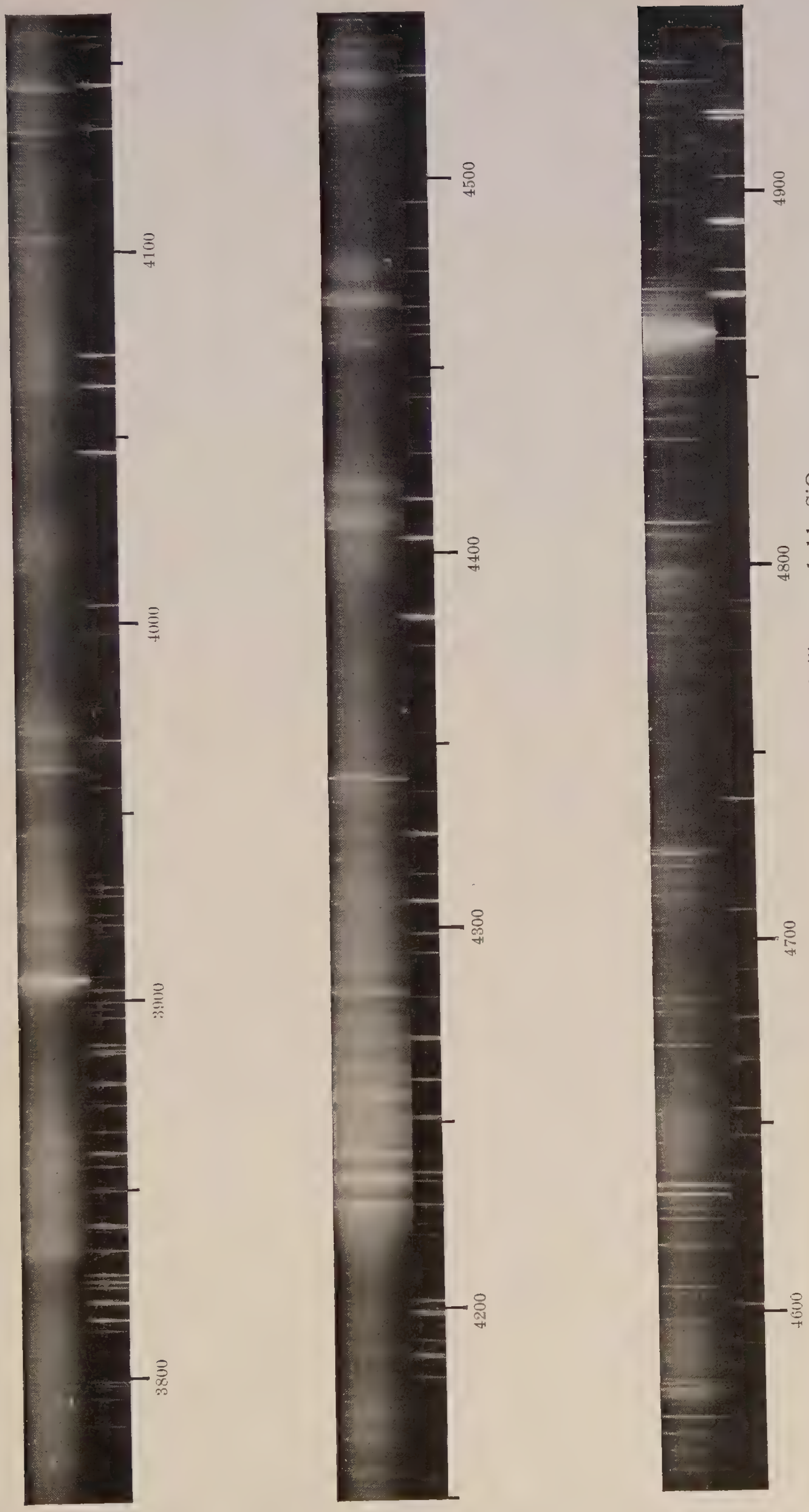

Band-system ascribed to an oxide of silicon, probably SiO_2.

To face p. 708.

discharge had been switched off, and the spectrum was developed best under conditions in which the silica constriction became hottest. After one series of exposures it was found that the constriction was punctured and that a clear deposit of glassy flakes had formed in a cooler part of the discharge tube.

That silica was being vaporized was further evidenced by the presence of a number of silicon lines in the spectrum: that at λ 3906 (SiI) is conspicuous in the plate reproduced here. The plates taken with the quartz spectrograph showed that bands in the ultra-violet region known to belong to SiO were developed at the same time, while the plates taken on the grating reveal also the SiH heads at $\lambda\lambda$ 4142 and 4128.

Other support is given in a note published in 1927 by Cameron (2) on two band-systems associated with silicon. One system was obtained from an arc between silicon poles in oxygen. Cameron observed that whereas the arc in air or oxygen at atmospheric pressure was whitish in colour and gave SiO bands, yet in a slow stream of oxygen at a pressure of about 10 mm. mercury the colour was an intense blue and gave, besides SiO, a complex system of five or six bands in the blue region, with the appearance of an isolated band at λ 4214, degraded to the red. The constituent bands occurred in pairs and were degraded to the red.

This description suggested that the two systems were the same. From a comparison (see table) of the measurements and intensity-distribution, and the similarity of appearance to the plate reproduced in his note, it appears fairly certain that Cameron's bands are a part of the spectrum now obtained with a discharge-tube source and photographed further to the violet. After having investigated possible impurities in the silicon poles, Cameron too concluded that the emitter was an oxide of silicon. The general complexity of the system now suggests that the excited molecule is polyatomic, perhaps SiO_2.

The other system reported by Cameron, obtained from a discharge through a mixture of silicon tetrachloride and excess oxygen, does not appear in the present source.

ACKNOWLEDGEMENTS

I should like to thank Dr. R. W. B. Pearse and Professor H. Dingle for their interest and helpful discussion of the paper.

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A SIMPLE METHOD OF FINDING POISSON'S RATIO

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Received 13 *June* 1940

ABSTRACT. The paper describes how Poisson's ratio can be measured in terms of the ratio of two specified natural frequencies of a vibrating free square plate. Results for glass, steel, brass, copper and aluminium are given. It is suggested that such a simple method of estimating this elastic constant might be of value in the mechanical testing of metals and alloys.

§ 1. PRINCIPLE OF THE METHOD

IMAGINE a free square plate made of a material which has a zero value of Poisson's ratio (1). The simple vibrating modes having the nodal systems 2|0, 0|2 shown in figure 1, *a* and *b*, would then be possible. These modes could combine, either in opposite or like phase, to produce also the 2|0− and 2|0+ nodal systems of *c* and *d*. The periods of the four modes would all be equal. In practice, since Poisson's ratio is never zero, and on account therefore of the anticlastic curvatures of the plate, the simple vibrations *a* and *b* can never be obtained singly but only in combination, as shown in the two photographs of figure 2. The natural frequencies of these two modes diverge from one another as Poisson's ratio increases, that of the 2|0+ mode being the greater.

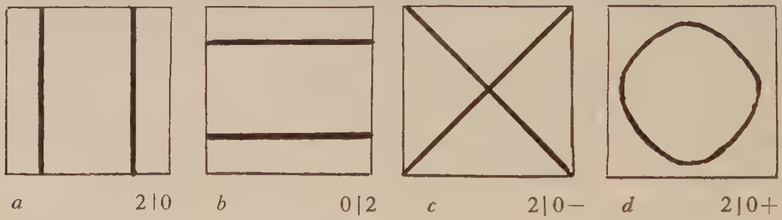

Figure 1.

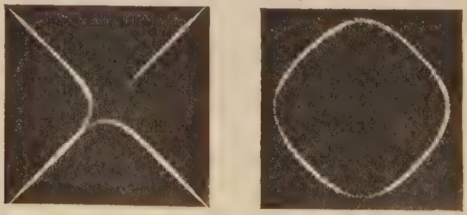

Figure 2.

The calculations of Ritz(2) and of Lemke(3) have been used to construct the two lines shown in figure 3, one of which gives the frequencies (in arbitrary units) of the $2|0+$, and the other the frequencies of the $2|0-$ mode, for different values of Poisson's ratio. The variation of frequency with Poisson's ratio is approximately linear(3), and the lines should intersect for a zero value of the latter quantity. Thus Poisson's ratio must be proportional to the difference between these frequencies, and if the relative frequencies are known, Poisson's ratio can be found.

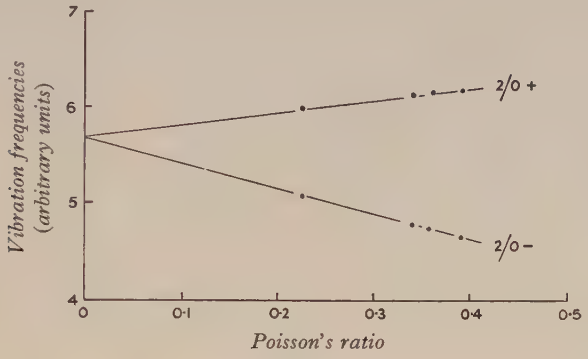

Figure 3.

It is convenient in practice to find Poisson's ratio from the interval or ratio between the frequencies of the $2|0+$ and $2|0-$ modes. This interval is plotted in figure 4 against Poisson's ratio, values having been taken from figure 3.

The above is a particular case of a method which might be applied to other modes, but consideration(4) shows that the dependence of frequency on Poisson's ratio becomes rapidly less as we pass to higher overtones.

§ 2. PRELIMINARY TESTING OF THE METHOD

This method of finding Poisson's ratio was tested for glass, steel, brass, copper and aluminium. The dimensions of the plates used (except number 6) are given in table 1, since, although it is unnecessary to know these, it is convenient, from the experimental standpoint, to have them recorded. Either bowing or the solid carbon dioxide method(5) of exciting free vibrations was employed to obtain the nodal systems, the corresponding frequencies being measured by means of a calibrated valve oscillator. Full experimental details have been given in previous papers(4), (5).

Table 1. Dimensions of plates

Specimen	1	2	3	4	5	7	8	9	10
Length (cm.)	20·4	8·14	20·4	5·10	20·3	5·11	5·10	20·5	5·08
Thickness (mm.)	3·08	1·39	1·94	1·50	4·88	1·64	1·58	2·02	1·66

The results of the observations are recorded in table 2, in which the 2|0+ and 2|0− frequencies of the various plates are given in the third and fourth columns respectively. The ratios of frequencies are contained in the fifth column, while the values of Poisson's ratio, as deduced from figure 4, are shown in the sixth column of the table. These last may be compared with the values of Poisson's ratio, taken from tables, given in the seventh column. Considering that the ratio is affected by the purity of the specimens, and that, for the metals, it depends also on the previous mechanical and heat treatments, the two columns of figures are in satisfactory accordance.

Table 2

Speci-men	Material of plate	Frequencies (c./sec.)		Ratio of frequencies	Poisson's ratio	
		2 | 0+	2 | 0−		From figure 4	From Kaye & Laby
1	Glass, crown	430	377	1·141	**·20**	·20 ·27
2	,,	1251	1091	1·147	**·21**	
3	Steel	267·5	231	1·158	**·22**	·25–·33
4	,,	3330	2660	1·251	**·32**	
5	Brass	502	390·5	1·285	**·36**	·34–·40
6	,,	2359	1840	1·282	**·35**	
7	,,	2677	2056	1·302	**·37**	
8	Copper	2714	2058	1·318	**·38**	·34
9	Aluminium	308	234	1·316	**·38**	·34
10	,,	3925	2967	1·323	**·39**	

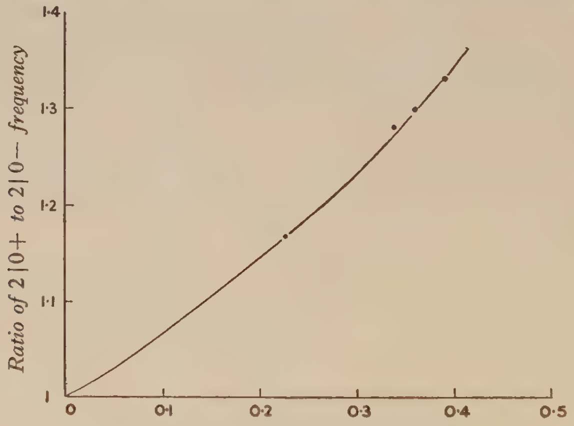

Figure 4.

§3. CONCLUDING REMARKS

In view of the results described above it would be interesting, when circumstances permit, to resume this study on specimens which could also be tested by an independent method. The square plates, for example, could be cut from such bars as were used by Ferguson and Andrews ⁽⁶⁾ in their study of anticlastic bending. At a time when the mechanical properties of metals and numberless alloys are receiving so much attention, such a simple method of finding Poisson's ratio might prove to be of practical value.

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REVIEWS OF BOOKS

Advanced Calculus, by I. S. Sokolnikoff. Pp. x+446. (New York and London: McGraw-Hill Publishing Co., 1939.) 26*s*.

The phrase " advanced calculus " is perhaps not very descriptive, or at least may be used in different senses by different writers. Here it covers an introduction to analysis, omitting the theory of the complex variable, so that though the calculus may be advanced, the mathematics is not.

After a chapter on limits and continuity, two are devoted to differential calculus, with such matters as the differentiation of implicit functions, and a paragraph on applications to solid geometry. Integration (entirely Riemannian) follows, and then come two chapters on infinite series, with the elementary tests of convergence. There is a chapter on Fourier series, and one in which the properties of the Jacobian determinant are introduced. As illustrating the scope of the book, we may note that the gamma function is treated (though the exclusion of complex theory is a considerable restriction), but there is little more than a definition of the elliptic integrals, and no reference to Landen's transformation.

Thus there is perhaps less detail in the book than in some of the older treatises on calculus, but what there is is treated in a much more rigorous—though not repellent—manner. Indeed, the style and format of the book are both as attractive as they were in the earlier work (*Mathematics for Physicists and Engineers*) by the present author and his wife.

J. H. A.

Advanced Calculus, by C. A. Stewart. Pp. xviii+523. (London: Methuen and Co., Ltd., 1940.) 25*s*.

Unlike the book reviewed above, the title " advanced calculus " attached to this one covers Lebesgue integration and complex theory, but in other respects its scope is somewhat similar. The treatment is much more difficult ; thus, as in the other work, the first chapter deals with sequences, but in this case it goes on to sketch briefly the Cantor definition of real number—a somewhat refined concept for a student at this stage. Again, before functional continuity is defined, there is a brief account of the theory of sets of points (necessary of course before Lebesgue integration can be considered).

An interesting and valuable chapter is on vectors and tensors, including such subjects as scalar and vector products of ∇ with a vector or scalar, and covariant derivatives of tensors.

In the chapter on complex theory, Taylor's theorem is given as well as Cauchy's ; there is a section on conformal representation, including the Schwarz-Christoffel theorem, and the evaluation of real definite integrals by contour integration is discussed and illustrated. Among special functions, the gamma function is treated in considerable detail, and the elliptic functions are defined as functions analytic except at certain poles, and doubly-periodic with periods whose ratio is not real. The elliptic integrals are mentioned as a sub-class of Abelian integrals.

The book is abundantly supplied with examples. The printing is good, and though the pages present an unpleasantly crowded appearance, the alternative was no doubt a considerably dearer book since, after all, a very great deal has been crammed into the space.

J. H. A.

Molecular Spectra and Molecular Structure: *I. Diatomic Molecules*, by G. HERZBERG; translated by J. W. T. Spinks with the co-operation of the author. Pp. xxviii + 592. (New York: Prentice-Hall, 1939.) $6.50.

In the notice of the original German edition of this excellent work, which appeared in these *Proceedings* (vol. 51, p. 558) sixteen months ago, it was mentioned that an English translation was being undertaken by Prof. J. W. T. Spinks of the University of Saskatchewan, who had already translated Herzberg's companion work on *Atomic Spectra and Atomic Structure*. It was also planned that the present volume on diatomic molecules would be followed by another, of which there would also be both German and English editions, on the spectra and structure of polyatomic molecules. Of all the German books on diatomic spectra none is more worthy of translation than Herzberg's excellent presentation of both the observational and the theoretical aspects of the subject. So admirably has Professor Spinks carried out this task that there is no trace of the German origin, and the result is perhaps the best English text that has yet appeared on this subject. The book is excellently produced ; only as regards the reproduction of the spectrograms does it compare at all unfavourably with the original.

In both editions those sections that are not essential to an understanding of the fundamentals, and that may therefore be omitted in a first reading by the beginner or the non-specialist, are printed in smaller type than the remainder of the text. Additions to, and improvements upon, the original treatment have been included in the present version, but the tabulation of constants derived from known diatomic spectra is still, unfortunately, confined to ground states. The very excellence of the book seems to make it the more regrettable that the tabulation has not been extended to include the constants for all the well-established excited states, even though such an extension would have required a good deal of space and a great amount of work. For collected data on excited states it is still necessary to refer to, and build up from, Appendix II of our Society's *Report* (1932), or the similar table in Volume I of Sponer's *Molekülspektren* (1935).

W. J.

Light and Colour in the Open Air, by M. MINNAERT; translated by H. M. Kremer-Priest, revised by K. E. Brian Jay. Pp. xi + 362. (London: G. Bell and Sons, Ltd., 1940.) 15*s*.

Professor Minnaert has written a kind of optical " Eyes and No Eyes ", which may have a good deal of influence on the teaching of " Light ". The biologists seem to be in the middle of a movement out of the school laboratory into the school farm and garden; if the author of this book had his way we might almost forsake the dim dark room (with its pins, prisms, and spectrometer) for the country ; visit a lake for a study of reflection and refraction; tour to the tropics in search of the green flash at sunset; and spend our nights in naked-eye observations on the stars. There can be few outstanding phenomena of light and colour in ordinary experience of Nature that are not noticed in this book. It is most clearly written and illustrated.

There is a fund of information, now gathered in these pages, which will answer some of the questions that have puzzled us. Probably most keen observers who have the necessary knowledge of fundamental optical principles will have already noticed many of the things discussed, but it will be a mine of good things for any teacher who has to teach optics without many resources in the way of special apparatus.

L. C. M.

Elementary Survey of Physics, by ARTHUR E. HAAS and IRA M. FREEMAN. Pp. x+203. (London: Constable and Co., Ltd., 1939.) 7*s*. 6*d*. net.

It is unnecessary to write at length here about this book, since it is based upon Professor Haas's *Elementare Physik* (though this fact is not stated in the preface or elsewhere), which was reviewed in these *Proceedings* (vol. 49, p. 633) just three years ago. The English title is the more apt, since the book is a concise and comprehensive survey of a wide field, rather than a text-book in the ordinary sense. It may, in fact, be heartily recommended to Higher School, Intermediate Science and First Medical students as a supplement to their usual text-books. Some of the later sections of the German edition, e.g. those on the Compton, Zeeman, Stark, and Raman effects and band spectra, have been omitted. Instead, there are two very useful and interesting Appendices with the titles " Physics and Civilisation " and "' Supplement for Pre-medical Students ". Into a third appendix are collected the proofs of some fundamental formulae which appeared in their appropriate places in the main text of the German edition. The object of this change is doubtless to make the text even more easily readable by those who are not mathematically inclined; the transposed material is so easy, however, that it might well have been left where it was —in smaller type if any distinction were desired. It is pleasant to note that the loose statements in the German edition do not appear in this.

W. J.

Sound Transmission in Buildings: Practical Notes for Architects and Builders, by R. FITZMAURICE and W. ALLEN (Dept. of Scientific and Industrial Research.) Pp. 48. (London: H.M. Stationery Office, 1939.) 4*s*.

This is a Report based on research which has been done in recent years at the National Physical Laboratory and the Building Research Station. The three parts deal with (1) the ways in which sound from the various types of source is transmitted, (2) the insulation necessary to reduce the noise to a level which will allow activities such as conversation, office work and music to take place without disturbance within the building, (3) actual types of discontinuous structure necessary for isolating rooms from each other in blocks of flats, semi-detached houses, etc. This last section will be of the greatest service to the builder, since it shows how the technique of floating floors and suspended ceilings may be applied in practice, with full details of the ventilation and plumbing systems necessary to carry out the principle of construction " in layers ". To the physicist those portions which deal with the types of vibration and their propagation through a structure will have most interest. The authors have found that flexural vibrations of the walls, etc., are of more importance in transmission than the compressional ones. This suggests a pinning of the walls to make the transmission of bending moments impossible or the use of edge fixings making the setting up of such moments difficult. The predominance of flexural vibrations makes the natural resonance of partitions low in pitch, and, unfortunately, low-pitched sounds are more difficult to isolate. It is emphasized that impact noise cannot be insulated merely by giving the structure mass or rigidity, and this is where the discontinuous constructions which it is the main object of the book to urge upon builders have the advantage.

E. G. R.

The Scientific Journal of the Royal College of Science, Vol. X. Pp. iv+135. (Royal College of Science, S. Kensington, S.W. 7, 1940.) Paper, 4*s.*; cloth, 5*s.* 6*d.*

This volume, which contains a selection from the papers read before the Imperial College Chemical Society, the Royal College of Science Natural History Society and the Royal College of Science Mathematical and Physical Society, provides remarkably fresh and interesting reading to the " specialist in other subjects " and, indeed, to the specialist in his own subject. Five papers are published in the Chemical section, two in the Natural History section, and four in the Mathematical and Physical section. All are clear and critical accounts of the state of existing knowledge in the topics dealt with; all are interesting; and if it be not considered invidious to pick and choose between the members of three excellent teams, attention may be drawn to the papers on " The Structure of Metal Films ", " The Crystallography of Black Carbon ", " The Biologist in the Tropics ", " Meteors ", and " The Properties of Liquid Helium ".

The editors are to be congratulated on providing such good and varied provand, and on their courage in deciding to continue publication, even in these days of grave stress. The volume is excellently produced, and the price is most modest. A. F.

Records and Research in Engineering and Industrial Science, by J. EDWIN HOLMSTROM. Pp. xii+302. (London : Chapman and Hall, Ltd., 1940.) 15*s.* net.

This book should prove useful, not only to scientific workers in many fields, but also to anyone who has scientific interests or whose work has a technical or scientific background. It deals with three aspects of technical knowledge :—(1) The production of knowledge, i.e., research, either pure or applied. (2) The storing and indexing of the knowledge thus produced. (3) The means which are, or might be, adopted to put into effective circulation the enormous mass of knowledge which is being acquired, and of which much, at present, becomes known only to a very limited number of people.

For those already engaged in scientific work, the chapters dealing with the second and third sections should prove most stimulating. The author's description of indexing and filing systems, especially the one developed by himself, will excite envy in those who have had to grope through dusty cupboards in search of a copy of some elusive pamphlet, and may even inspire some to achieve the same degree of orderliness and, mayhap, of resultant happiness.

The chapters dealing with abstracting and bibliographical services are really invaluable, and even a moderate use of the facilities described would save much wasted effort on the part of many students and research workers. The author is to be commended for the very complete information which he has given regarding them. It is one of the tragedies of the war that many of these facilities are no longer available, and it is to be hoped, when normal conditions are restored, that some of the suggestions of international co-operation in this important service may be taken up.

Among other matters dealt with are note-taking, translating and technical dictionaries. The author writes from personal experience of the systems and methods described. One may not agree with everything he says, but one can agree whole-heartedly with the subtitle of the book—A Guide to the Production, Extraction, Integrating, Storekeeping, Circulation and Translation of Technical Knowledge. J. G. D.

The Photographic Process, by JULIAN ELLIS MACK and MILES J. MARTIN. Pp. xvii + 586. (New York and London: McGraw-Hill Publishing Co. Ltd., 1940.) 33*s*.

In the preface to this new book on photography the authors remark that, despite the immense growth in general interest in the subject during recent years, surprisingly few books have appeared on it written from a scientific standpoint, So while preparing a book intended primarily to serve as a text for students taking the course in photography at the University of Wisconsin, they have endeavoured to design it in such a way that it may appeal to other readers who desire to make their practice of photography "an intelligent application of principles rather than a blind following of rules of thumb". The result is a fairly rigorous, up-to-date, well-balanced survey of the photographic process and its applications, which nevertheless retains a strong flavour of popular appeal.

The emphasis being on fundamental principles, the book is naturally devoted to basic technique rather than to studio craft, and therefore detailed description of such practices as retouching and portrait lighting are excluded. After a brief historical introduction, nine chapters follow in detail the main process from original subject to finished print by dealing in turn with optics, lenses, cameras, sensitive material, exposure, development, colour sensitivity and filters, printing and enlarging. The remaining four chapters describe the application of the process to natural-colour photography, to scientific and technological photography, to photomechanical reproduction and to pictorial photography respectively. Illustrations for the chapter on scientific and technological applications are drawn from fields as widely separated as astronomy, spectroscopy, medicine, industry and war, ranging from a stereoscopic cloud-chamber photograph of the end of a meson track to the photographic detection of camouflage.

The book throughout is well furnished with interesting and attractive examples of the photographic art. At the end the authors have added an extensive bibliography, appendices on chemistry and mathematics applicable to photography, a list of formulae and a course of practical work.

A useful book. R. W. B. P.

Electronic Structure and Chemical Binding, by O. K. RICE. Pp. xiv + 511. (New York and London: McGraw-Hill Publishing Co. Ltd., 1940.) 33*s*.

Many physicists and chemists go through life believing that anything can be proved by thermodynamics; soon a similar misconception may be prevalent about quantum mechanics. Both subjects are fundamental for a proper understanding of physics and chemistry, and both develop their arguments mainly in a mathematical language. There is, however, a difference in the kind of difficulty which they present. Briefly, the difficulty in thermodynamics is to understand some mathematics which is easy to perform; in quantum mechanics, it is to perform some complicated calculations the purpose of which is clear. The incomprehensible simplicity of the mathematics of thermodynamics must exasperate many readers, or fill them with despair. Fortunately in quantum mechanics the situation is not so trying. Here it is possible to work out from beginning to end with clarity a few simple examples, to generalize widely on these, and then to discuss a fascinating range of applications. Prof. O. K. Rice has just produced a book along these lines, choosing as his applications those of inorganic chemistry.

The book is divided into nineteen chapters of roughly equal length, five appendices, author and subject index, and a page index to molecules and crystals mentioned in the text. The first three chapters are of the usual introductory type, about electrons, x rays, e/m, etc. All books seem to have these; is it not time that one book or monograph did this introductory work so well that it was finished once and for all? The next half-dozen chapters discuss the energy levels and wave functions of an electron in a box, the plane and space rotators, and the hydrogen atom, electron spin and many-electron atoms, including the periodic table and ionization potentials. Now follow chapters on molecules. These deal with molecular energy curves and normal modes, the various mathematical theories of the hydrogen molecule, the pair and orbital methods for more complicated molecules, and a discussion of bond energies and the electro-negativity scale. The remainder of the book, roughly one-half the length, covers an enormous range of applications. Thus—to choose at random—short, pithy accounts are given of directional properties of bonds, coordination numbers, covalent and ionic radii, the Born-Haber cycle and applications, ionic, molecular and metallic crystals, as well as a whole chapter about water and aqueous solutions.

The character of the book can perhaps be best conveyed by stating that there are no fewer than 55 tables, all of which contain data in constant use by chemists. Another excellent feature is that great care has been taken to explain the experimental evidence on which the theory is based, even though the theory itself is only briefly described. The dynamical principles on which many of the applications rest are, of course, those of quantum mechanics, but the dynamics are so skilfully woven into the whole pattern that one is able to forget the quantum mechanics and to regard the application as a part of chemistry.

Professor Rice has undoubtedly produced a book that will save others much labour. Many ready-made lectures in the field of theoretical chemistry, amply supplied with the most recent experimental figures, are there in highly polished form. The diagrams are models of effectiveness combined with simplicity. Perhaps the mathematics is cut down too much, but to some students this is an added advantage. W. G. P.

Chemical Spectroscopy, by WALLACE R. BRODE. Pp. xi + 494. (New York: John Wiley and Sons; London: Chapman and Hall, Ltd., 1939.) 36*s.* net.

As its title implies, this book is not mainly concerned with the physicist's study of line and band spectra for the verification of spectral theory and the determination of atomic and molecular energy levels and constants, but is a text-book for metallurgists, chemists and bio-chemists who are engaged in research and control laboratories of the heavy chemical and dye industries, or in medical and biological research or in academic instruction and research. Indeed, the author may justly claim to have provided within a single volume two overlapping courses of instruction: (*a*) in qualitative and quantitative spectrographic analysis (chapters I–V and XII–XIV), and (*b*) in the observation and interpretation of absorption spectra of organic and inorganic compounds (chapters I, III, VI–VIII and XII). Some portions of the book have grown from mimeographed instruction sheets used in Professor Brode's course at the Ohio State University, and other portions from reports prepared by him for several committees.

After an elementary sketch of the theory of atomic and diatomic spectra (chapter II), there follow eight chapters on emission and absorption spectra, qualitative and quantitative analysis, resonance and chemical structure, infra-red and Raman spectra, and colour. The next is a long chapter (XII) consisting of notes and instructions on a series of twelve laboratory exercises and a lecture demonstration. Then come two short chapters on photography and on the equipment of a spectroscopic laboratory for work of the kind dealt with in this book.

The essentially practical nature of the book is well seen from this outline of its contents, and is further emphasised by the large numbers of line diagrams and photographs of instruments and accessories and of reproductions of spectrograms and absorption curves in the text, by an 119-page appendix of tables of numerical data (chiefly wave-lengths of atomic lines), and by a packet of such supplementary material as cellophane colour filters, film negatives of spectrograms, etc., which is supplied with the book. Mention should also be made of a series of novel charts of the iron-arc spectrum from 2310 A. to 5090 A. at the end of the book: on each chart the wave-lengths of the Fe lines are given on one side and those of prominent lines of other elements on the other side, but between the spectrogram and the latter is a blank area on which a spectrum under examination may be projected for the purpose of identification of atomic lines and estimation of wave-lengths.

The book is adequately indexed, is excellently produced, and should be very useful to the chemical readers for whom Professor Brode has amassed such a wealth of material in comparatively little space. Three notes may be added here for the author's consideration in the preparation of a second edition: (i) fewer photographs of apparatus, especially exteriors of instruments from makers' catalogues, should be reproduced; (ii) the behaviour of the rays in figure 12.44 *a* is not above suspicion; and (iii) the slits used for many of the charts on pp. 448 to 482 appear to be unduly wide—e.g. the λ3100 triplet shows hardly any sign of resolution on p. 467 although it is clearly resolved on p. 69. W. J.

Electrical Communication, by A. L. ALBERT. Second edition. Pp. ix + 534. (New York: John Wiley and Sons; London: Chapman and Hall, Ltd., 1940.) 30*s*. net.

The second edition of this textbook for students of electrical communication succeeds in covering the very wide field of the subject, including in two of the opening chapters much on speech, hearing and general acoustics. The fundamentals of the subject are dealt with concisely, and generally admirably, early in the book and in two later chapters devoted to transmission theory and networks. The remainder of the book is devoted to such of those applications of these fundamentals to the production of a complete telephone system as have been developed by the Bell Telephone System.

Either the author is not well acquainted with many of the applications of the subject made by Dutch, German and British physicists and engineers or he has deemed it better to ignore all references to such. The slavish adherence to the American system makes the book specially useful for any short-sighted American students or for those British readers who wish to know how various aspects of the subject are treated in practice in America. The mistake, commonly made, of attempting to describe, simultaneously, several different systems has been avoided, and one does get a good general idea of American Telephony, Telegraphy, Cable loading and the design of much auxiliary apparatus.

A chapter is devoted to inductive interference and plant protection in which the methods of reducing the cross-talk between two or more telephone circuits is given only a passing mention, whereas power interference is treated very fully. In practice, however, cross-talk is as serious a problem and has received as much attention as power interference. Electronic devices and their uses are dealt with moderately adequately in two following chapters. Readers in this country will note the lack of reference to the valveless echo suppressor, systems for the transmission of dialling impulses over long trunk circuits and certain well-known features of European telephone systems. The author has been unable to find space for reference to the instruments used in electrical communication, a knowledge of which is of great importance to every student.

A clear, concise style is used throughout the book and the diagrams are good, though standard symbols are not always used. The book should be read and studied by all keen students and by engineers already engaged in the subject. J. R. T.

Sound, by E. G. RICHARDSON. Third edition. Pp. vii+339. (London: Edward Arnold and Co., 1940.) 16*s*. net.

There was a time in the history of physics when an author having completed a book on sound could sit back with such complacency as he could muster, knowing that the subject would, so to speak, "stay put", and that any question of new editions would make few further demands on him. But those days have long passed, and the development of applied acoustics, in particular, has been such that of late years an author is likely to have the uncomfortable feeling that his book is already "dated" before it reaches publication. The task of revision is probably less onerous in the case of a textbook such as the one under review, and Dr. Richardson has made a valiant and not unsuccessful effort to keep his book, first published in 1927, in the forefront of progress. As he himself points out, the present position in sound is very different from that when his book originally appeared: there is now a spate of works dealing with new developments of the subject, so that the student is no longer embarrassed in his efforts to find authoritative treatments of the various aspects.

The reviewer, in welcoming the new edition of a book which has already won its spurs, notes that in a work which deals adequately with musical instruments and musical scales there appears to be no mention of the different concert pitches adopted in the past, or of the international standard of concert pitch (A=440 c./s.) adopted at an international conference in London last May.

Physics for Medical Students, a Supplementary Text-Book, by J. S. ROGERS. Second edition. Pp. xiii+304. (Melbourne University Press in association with Oxford University Press, 1939.) 12*s*. 6*d*. net.

As every teacher of first-year medical students knows, it is quite impossible, in the time available, to make the physics course all that it should be. He can, and does, follow up his presentation of each fundamental principle with a brief mention of some example or application which will be met either in another part of the course for the medical degree or in medical practice afterwards. But the course remains just a modified intermediate, differing less than it should differ from the intermediate for the science or for the engineering degree. And so it must remain until far more time can be allotted to physics, and until first-year medical students are everywhere taught quite separately from students in other faculties, as, of course, they already are taught in, for example, the medical schools of many of the large London hospitals.

This supplementary textbook presents much of the additional material that would be included in the desired course. The first edition was the outcome of a first-year medical students' pamphlet prepared by Dr. Hercus and Professor Laby to supplement the ordinary textbooks. To many in our English medical schools the first edition is already well known, and it is only necessary to say that this second edition contains new chapters on such topics as surface phenomena, the electrocardiograph, measurement of body temperature, plane polarized light, nuclear transformations, etc., as well as the necessary revisions and extensions of the old chapters. There are important sections, such as those on the therapeutic use of X and γ rays, methods of producing high potentials for X-ray tubes, apparatus for purification of radon, etc., which are provided for reference but are not required for the first-year examinations. It is clearly a book that should be in the hands of all teachers of medical students, as well as of the students themselves, and of qualified medical men, especially those engaged upon physical therapy. W. J.

A Text-Book of Heat: *Part II*, by H. S. ALLEN and R. S. MAXWELL. Pp. ix+317. (London: Macmillan and Co., Ltd., 1939.) 10*s*. 6*d*.

Part I of this book was reviewed last year (vol. **51**, p. 1050 of these *Proceedings*) and was intended to cover the ground of the Higher School Certificate or Intermediate Examination in Physics. Part II is of a more theoretical and advanced character. The two volumes taken together will meet the requirements of those reading for a pass degree in physics.

The subject is developed along the historical path; from speculations of the early philosophers on the nature of heat through the conceptions of an igneous fluid in the 18th century to the dynamical theory of the 19th century, and, finally, to the more abstruse ideas connected with the quantum theory. Following on the first law of thermodynamics a critical survey is given of the various investigations which have been carried out to determine Joule's equivalent. At one time the accurate determination of the value of J was looked upon as a matter of fundamental importance, but the position has changed very considerably in recent years. Calorimetric apparatus is usually calibrated electrically and the necessity for high accuracy in the value of J has disappeared. It is, however, most desirable that some definite value should be agreed upon. It might be noted in passing that the Steam Table Conference has proposed that the International Kilogramme calorie should be taken as 1/860 international kilowatt-hour.

Chapters are devoted to transport phenomena in gases, entropy, and the approach to the absolute zero. In the last-mentioned there is an account of the heat theorem of Nernst. Optical pyrometry is concisely and clearly explained, though the authors should have added that the extension of the scale of the disappearing type of instruments intended for industrial work is effected by the use of neutral glasses for cutting down the incident radiation. The rotating sector which is referred to is confined to the Laboratory Standard type of instrument. Later, when dealing with the polarizing type of optical pyrometer, the authors mention absorption glasses for extending the scale of these instruments. The chapter on the quantum theory of radiation is an admirable résumé of the subject, whilst the chapter on statistical methods and probability leads on to Maxwell's distribution law and to entropy and probability. In the final chapter the authors consider the modifications brought about through the introduction of the quantum theory and therein we find brief accounts of the Bose-Einstein and the Fermi-Dirac statistics. This is to form an introduction to such treatises as Fermi's *Thermodynamics* and Guggenheim's *Modern Thermodynamics by the Methods of Willard Gibbs*. There is no doubt that the student who masters these two volumes will have had a good grounding in physics, and the authors are to be congratulated on the results of their labour. E. G.

Science since 1500. *A Short History of Mathematics, Physics, Chemistry and Biology*, by H. T. Pledge, B.A. Pp. 357, with 15 plates. (London: His Majesty's Stationery Office, 1939.) 7*s.* 6*d.*

This book, according to the preface, was written to serve as a background study to the manuals on the history of special scientific subjects prepared from time to time by the Science Museum. It is intended for the " specialist in other subjects " the scholarship candidate, the university student and the research worker.

This is a rather ambitious aim and must have involved a most formidable amount of painstaking preparatory work. Although the author exhibits a vast knowledge of the subjects concerned, combined with considerable literary skill in compressing the whole of the history of science since 1500 into about 300 pages, the book is somewhat disappointing. The main cause of this is the inclusion of too much of the mass of detail which constitutes scientific knowledge, at the expense of explanatory matter. Somewhat more discrimination in the selection of material than is shown by the author is required for a background study.

The result is that we are presented with interesting but very technical, well-written reviews of the historical development of the various sciences interspersed with a few chapters dealing with special topics or with more general historical aspects of science as a whole.

The chapters dealing with physics and related subjects occupy more than one half of the book and provide the most complete historical outline of their development which has yet appeared. The whole of the book requires considerable existing knowledge of the various subjects on the part of the reader. It is doubtful, therefore, if the physicist will, in general, find the chapters dealing with the other sciences as satisfactory as those which deal with his own. H. B.

The Cyclotron, by W. B. Mann. Pp. x+92. (London: Methuen and Co., Ltd., 1940.) 3*s.*

The appearance of a monograph on the cyclotron will be welcome to many readers who have a general interest in physics, as well as to professional physicists. Dr. Mann's book gives an interesting and lively account of the development of the cyclotron and a fairly detailed description of the apparatus built by the inventor of the instrument, Professor E. O. Lawrence, in Berkeley. He devotes one chapter to the magnet and to the vacuum chamber, with its auxiliaries, in which the acceleration takes place, one to the very important question of the high-frequency apparatus for supplying the accelerating voltage and another to some questions of the operation and adjustment of the cyclotron. In certain matters the author is inclined to be dogmatic, but perhaps not without some justification.

In some respects the book seems to lack perspective. Not every general reader will be familiar with the properties of protons, deuterons, etc., and even the physicist would probably benefit by some discussion of the various methods used for the acceleration of ions and their application to experiment in nuclear physics ; while no other apparatus can compare with the cyclotron in the production of ions of very high energy, there are nevertheless many types of experiment for which other methods have distinct advantages.

Apart from these matters there is little to criticize, and Dr. Mann's book is a very useful addition to a well-known series of monographs on physical subjects. J. C.

Magnetism and Very Low Temperatures, by H. B. G. Casimir. Pp. x+95. (Cambridge: The University Press, 1940.) 6*s*.

In this tract, based on a short series of lectures given at Cambridge two years ago, the author has a subject with which he is particularly well qualified to deal. The magnetic properties of paramagnetic salts, which are here exclusively considered, at temperatures below 1° K., are of much more than specialist interest. They are fundamental to any investigation in the very low temperature range, for on them depend the present methods for both the production (by the adiabatic demagnetization technique) and the measurement of the lowest attainable temperatures.

After a summarizing introduction, successive chapters deal with classical theory and experimental methods for measuring susceptibility, the quantum theory of paramagnetism, experiments on adiabatic demagnetization, the more detailed theory of low-temperature paramagnetism, experimental results with different salts, and relaxation phenomena.

Owing to the high susceptibility of paramagnetics at low temperatures, measurements may be made by a wide variety of experimental methods, including a number essentially similar to those ordinarily used for ferromagnetics. A useful classification of the main methods is given, and there are brief descriptions. As for ferromagnetics, demagnetization effects are important, and the awkward problems connected with the shape of specimens, and the packing of powders, are illuminatingly discussed.

It is stated that the thermodynamic problems involved " can be treated by standard methods and no special difficulties arise ". It is perhaps owing to this that an occasional looseness has crept into the thermodynamic arguments leading to standard conclusions. The rigorous atomic theory of paramagnetism does raise many difficult problems, and the account given of the relevant parts of the theory is excellent, in view of its brevity. A considerable familiarity with the general theoretical background is, however, required for a proper appreciation of the discussions of the effect of the electric field of surrounding ions (the crystalline field effect) and of magnetic interaction on the energy levels, and of their influence on susceptibility and specific heat.

A list of about a hundred references to recent work is included. In providing an authoritative survey of investigations in a field in which rapid progress has been made in the last few years, this addition to the Cambridge Physical Tracts fulfils admirably one of the main aims of the series. E. C. S.

The Musical Ear, by Ll. S. Lloyd. Pp. ix+87. (Sir Humphrey Milford at the Oxford University Press, 1940.) 6*s*. 6*d*.

This book is a collection of essays, a number of which have been published in musical journals already, on various aspects of musical acoustics. The author possesses a flair for explaining the mathematical and physical aspects of sound in language which the average musician can understand, and the present work ably keeps up the standard of his previous works in this field. The leit-motiv running through the collection is the necessity of appealing to the ear as judge in all sound estimation. Musical quotations are used in place of talk of frequencies and decibels. The style is pleasant, and even the physicist with no musical pretensions will get pleasure from reading these essays.

E. G. R.

Electronic Processes in Ionic Crystals, by N. F. Mott and R. W. Gurney. Pp. xii + 275. (Sir Humphrey Milford at the Oxford University Press, 1940.) 20*s*.

The authors of this book have performed a double service to physics. First, they have collected together and coordinated a vast amount of experimental data on electronic processes in ionic crystals. Second, they have described, more or less " out of their heads ", a theory, based on quantum mechanics, to account for the observed facts. No recondite mathematics is used in their exposition; indeed, those familiar with the theory of metals will be able to start their reading anywhere in the book. Even those interested only in the experimental side will be able to glean much information with little effort.

There are eight chapters in all, an appendix, a name index and a subject index. Each chapter is divided into sections, usually nine or ten to the chapter, and the chapter titles and section titles are listed at the beginning of the book. The printing, the diagrams and the quality of the paper are of the uniform excellence of the International Series of Monographs on Physics, published by the Oxford University Press.

The first chapter is on the perfect ionic lattice, and the second is on lattice defects. Both are terse without losing clarity, and may be recommended to those wishing for a brief account of the classical theory of ionic crystals, largely associated with the name of Born, together with the crucial new points introduced by the disorder concept. Chapter III is on the energy and absorption spectra of electrons in polar crystals. The existence of any irregularity in the crystal, such as a crack or a foreign atom, may cause an electron to be trapped in the neighbourhood of the imperfection. This effect is discussed, and various possibilities for the energy spectrum of a trapped electron are worked out. Chapter IV describes the experimental work on colour centres in the alkali crystals and the photoconductivity in certain insulating crystals, such as diamond and the silver halides. Not much in the way of theory is attempted here except " order of magnitude " calculations, because at present no adequate theoretical treatment seems possible.

Chapter V is excellent. It is an essay on semi-conductors, oxide rectifiers and insulators, and has obviously profited from earlier accounts in other books. Since the peculiar electrical properties of a semi-conductor are due to impurities, either foreign atoms or stoichiometric excess of metal, the method of preparation of the semi-conductor may have an enormous effect on the electrical behaviour. Only by careful study of the experimental data can a theory useful to the practical man be developed. This chapter never leaves the experimental side except to establish a logical mathematical consequence from the observations. The models used for the theory are the usual one-dimensional straight-line barriers, wells and troughs, and they appear to be all that are necessary.

The rest of the book enters a region that terrifies most mathematical physicists. Whether the proposals made by the authors are likely to stand up to criticism in the future, the reviewer is unable to say, but even if this is not so, a very valuable contribution to this subject has been made by setting out the experimental facts in a way that the mathematicians can easily digest, and simultaneously proposing, with the aid of numerous diagrams, suggestions of the probable physical mechanisms of the processes. Chapter VI is on luminescence and the dissipation of energy in metallic phosphors. The next is on the photochemical reduction of silver halides, and the photographic latent image. The last chapter is about the rate of growth of tarnish films on the surface of metals.

This book will probably be the standard, and only, one in its field for many years to come, and, as such, should enjoy good sales. W. G. P.

Intermediate Practical Physics, by T. M. YARWOOD. Pp. xii + 307. (London: Macmillan and Co., Ltd., 1940.) 6*s*. net.

The distinctive character of this volume should ensure its welcome among the growing number of recent publications intended for students who are preparing for the Intermediate, Higher School Certificate and various University Scholarship examinations. The assumption that the student has a previous knowledge of the subject enables the author to dispense with some of the standard elementary experiments, notably those belonging to the sections of Mechanics, Hydrostatics and Heat, thus making room for additional matter. This assumption also gives him latitude in the order of presentation. We may note the inclusion of continuous-flow calorimetry, Stefan's fourth-power law, Newton's rings, the electric vibrator for measuring the frequency of the alternating supply, the use of the quadrant electrometer, and thermal emissivity. Several methods of verifying the inverse square law in magnetism and of comparing the capacities of condensers are also given. It is satisfactory to find mumetal suggested in the susceptibility experiment; the heading " Susceptibility of *iron* " is an evident slip. A fragment of solid carbon dioxide would be more convenient than the cylinder mentioned in connection with the measurement of sound velocity by means of a resonance tube. The author has devised an apparatus, which can be purchased, for the plotting of resultant magnetic fields. The *Real-is-Positive* convention is adopted in the Light section and the use of ray-track apparatus, wherever possible, is advocated. No mention is made of latent heats, melting points, vapour pressure, photometry, or thermionic valves.

In the introduction there are good sections on errors of observation, accuracy of measurement, calculation of results and graphical work. Although, in the preface, the student is asked to approach his work in the spirit of a pioneer who " has arrived now at a stage when he is doing experiments on his own, relying largely on his own ability, ingenuity and initiative", he actually receives a maximum amount of assistance, since, not only the method of tabulating his observations but also graphs are given in connection with many of the experiments. The diagrams are excellent and an unusual amount of useful theory follows the clear descriptions of the apparatus required and the method to be used in each experiment.

The book concludes with a useful collection of physical data and mathematical tables which might be improved by the inclusion of tables of reciprocals. In view of the number of subjects dealt with, an index would have been welcome. The book can be thoroughly recommended to teachers of physics with whom it is likely to supplement rather than to supplant the courses of elementary practical instruction already in use. M. D. W.

Massachusetts Institute of Technology Wave-length Tables, compiled under the direction of GEORGE R. HARRISON. Pp. xxviii + 429. (New York: John Wiley and Sons; London: Chapman and Hall, Ltd., 1939.) 90*s*. net.

The main tables of this excellent volume form a catalogue of data on over 109,000 of the more prominent lines between 10,000 A. and 2000 A. in the emission spectra of neutral and singly-ionized atoms. In addition, nearly 1400 frequently occurring band heads have been included for identification purposes. Although, within this range, the lines listed number only about a half of all the lines known, they account for about 99 per cent of the energy emitted by atoms. With a few exceptions all known atomic lines having intensities of two or more on the chosen scale of 1 to 9000 are included. The reasons for the exceptions are given, with many other particulars about the plan of the work, in an admirably written and exceedingly interesting introduction by the compiler, whose statement of the object of the work is very modest; the catalogue, which was originally compiled for use in the M.I.T. laboratory only, is presented not as a finished product but as a preliminary compilation designed to improve the status of wave-length measurement and line assignment. That it will easily achieve this object goes without saying.

For some 75,000 of the lines catalogued the wave-lengths are taken from the first three years' results of the vast wave-length project which is being carried out by the staff and students of the spectroscopy laboratory of the M.I.T. under the notoriously able and inspiring leadership of their professor and with the well-organised assistance of the Works Progress Administration. For the remainder of the lines, the wave-lengths are taken from determinations by previous investigators and are considered to be the most trustworthy of those available in the literature of the subject; these comprise some 14,000 measured only in discharge tubes (since the M.I.T. work has been mainly confined to arcs and sparks), some 13,000 interferometer determinations (including International secondary standards), and some 15,000 other measurements which either are regarded as better than the M.I.T. results or belong to lines not certainly identified on the M.I.T. plates.

For the assignment of each listed line to a particular element the responsibility rests entirely with previous investigators, except in the cases of a number of lines of short wave-lengths (2400 A. to 2000 A.) not previously published; for many of these lines only tentative assignments to elements have been possible, but the data are included because thousands of strong lines not hitherto studied have now become observable in this region through the recent great increases in the reflecting power of gratings and in the sensitivity of emulsions.

Five large spectrographs, with gratings (ruled by Prof. R. W. Wood at Johns Hopkins University) specially selected and suitably mounted for photography in particular ranges of wave-lengths, are employed in the M.I.T. project, and up to the time of the compilation of the present catalogue more than 10,000 spectrograms had been taken in these instruments. Most of these were measured, not only by hand setting, but also at least twice on automatic and recording comparators one of which is shown, with a typical cinema-film record, in the frontispiece. Within the space of this review it is quite impossible to do justice to the accounts given of this or any other aspect of such a carefully executed piece of work, or of the discussions of such matters as sources of error and precision of wave-lengths. Tables of sensitive lines (arranged in alphabetic order of the elements and in order of wave-lengths) and a bibliography are also given in the introductory section. The compilers have been very wise in refraining from adding to the size of an already formidable volume by including wave-numbers as well as wave-lengths and from delaying publication until as many as possible of the listed lines had been tested by means of the combination principle. W. J.

Elasticité et Photoélasticimetrie, by H. LE BOITEUX and R. BOUSSARD. Pp. iv + 361. (Paris: Hermann et Cie, 1940.) Price 180 *fr.*

The phenomenon of photoelasticity depends so directly on the fundamental principles of both elasticity and the propagation of electromagnetic waves through crystalline media that any treatise on the subject must include at least an outline of both these basic theories. This exposition was so well made by Coker and Filon that the authors of the present volume must count it a compliment that the reviewer should remark that their presentation follows much the same general lines. Their treatment is perhaps slightly shorter, but no important detail is omitted, and certain practical details, as for instance the importance of a proper choice of material, are very well explained. Excepting a passing reference to the recent work of Oppel on the "freezing" of photoelastic effects by thermal treatment for the investigation of three-dimensional stress-systems, the book is concerned entirely with plane stress systems ; and all the various techniques by which such systems can be examined are adequately described. Only two examples of application of the method are developed in detail, but one is used to illustrate the utility of the method in design.

The printing and general presentation are good, but there are one or two instances of careless proof-reading, particularly in respect of lack of correspondence between diagrams and explanatory matter. The binding is, of course, very bad. H. L. C.

Contributions from the Physical Laboratories of Harvard University for the year 1938. Series II, Vol. V. Pp. vii+412. (Cambridge, Mass.: Harvard University Press; London: Sir Humphrey Milford at the Oxford University Press, 1940.) 21*s*. 6*d*.

This interesting collection of articles contributed to the periodicals of the learned societies during 1938 forms an impressive monument to the industry and enterprise of the department of physics at Harvard University. Into this volume there have been collected together reprints of forty-five papers dealing with different subjects, of too great a variety to permit of individual mention within the scope of this review.

In reviewing a volume of this kind it is possible to do little more than to analyse its contents, and such an analysis will perhaps be most concisely achieved by giving the names of the authors, an indication of the scope of their contributions being given by the subsequent list of periodicals to which the articles have been contributed. The numerals given in the following analyses denote the number of articles contributed by each author, some conjointly, or the number reprinted from any particular periodical.

Contributions have been made by the following authors: Bainbridge, K. T. (1); Bancroft, D. (2); Bardeen, J. (3); Breit, G. (1); Bridgemen, P. W. (6); Chaffee, E. L. (2); Chipman, R. A. (1); Cole, R. H. (2); Dwyer, R. J. (1); Edson, W. A. (1); Feenberg, E. (1); French, C. S. (1); Furry, W. H. (1); Griffin, D. R. (1); Hunt, F. V. (3); Hickman, R. W. (1); Jacobs, R. B. (3); Larsen, E. S. (1); Leighton, H. W. (1); Mah, Dah-You (1); Mimno, H. R. (1); Nier, A. O. (4); Oldenberg, O. (2); Pierce, G. W. (1); Pierce, J. A. (3); Purcell, E. M. (1); Rieke, F. F. (2); Schneider, E. G. (1); Smith, N. D. (1); Starr, C. (1); Stehn, J. R. (1) Street, J. C. (1); Van Vleck, J. H. (4).

The articles are reprinted from the following periodicals: American Journal of Science (2); American Physics Teacher (1); Electronics (1); Journal of Applied Physics (3); Journal of Chemical Physics (9); Journal of Mammalogy (1); Journal of the Acoustical Society of America (3); Journal of the American Chemical Society (1); Mechanical Engineering (1); Metals Technology (1); Philosophy of Science (1); Physical Review (14); Proceedings of the Institute of Radio Engineers (2); Review of Scientific Instruments (4); Science (1).

The chief appeal of a volume of this kind is essentially domestic, the articles in question having already been made available to the research physicist through the medium of the publications of the learned societies. For those who have been associated with Harvard such a volume must be of great sentimental value, but for the physicist in general and the specialist in particular it may prove to be redundant. It should prove, however, to be a welcome and a useful addition to the bookshelves of the libraries of scientific institutions.

W. B. M.

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Report on the Teaching of Geometrical Optics (1934). Price: 6*s.*

Report on Band Spectra of Diatomic Molecules. By W. JEVONS, D.Sc., Ph.D. (1932). Price: in cloth, 20*s.* 6*d.*; in paper, 17*s.* 6*d.*

The Decimal Bibliographical Classification of the Institut International de Bibliographie (1926). By PROF. A. F. C. POLLARD. Price : in cloth, 7*s.* 6*d.*

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